# Annex I
(informative)

# Conformance testing for SRF operations

## I.1 Introduction

This annex provides guidelines that may be useful for developing conformance requirements and conformance tests for implementation of the concepts specified in this International Standard including, but not limited to, the API specified in Clause 11.

## I.2 Computational error

The meaning of “error” depends on the context and application domain. Potential sources of error in SRF operations include formulation error, numerical approximation error, round-off error, truncation error and other errors associated with implementing SRF operations. In Annex B, computational error is defined to be the sum of digitization error, and those approximation errors made to simplify the implementation and/or to improve the computational efficiency of the process. Errors of this nature should not be confused with errors arising from modelling the true shape of a spatial object (celestial or abstract) by an approximation of the shape. In this International Standard, an ORM used to approximate the shape of an object is assumed exact. How well an ORM approximates the shape of a celestial object is outside the scope of this International Standard.

The specification of an SRF operation defines the domain and range, as well as providing a functional specification of how each value in the domain is converted into a value in the range. The functional specifications are the mathematical functions in one or more variables given in Clause 10. These functional specifications include a set of rules related to the appropriate ORMs, CSs, and bindings to the CSs.

## I.3 SRF operations baseline

Each SRF operation specified in Clause 10 has a theoretically exact specification in terms of mathematical functions. These formulations are specified assuming the use of theoretically exact arithmetic (infinite precision) for developing values of an SRF operation. These exact specifications fall into one of four basic categories:

a) a finite sum of elementary mathematical functions,

b) a finite sum of quadratures,

c) an infinite iterative process, or

d) an infinite power series.

In practice, implementations that use one of these categories require the use of finite precision arithmetic along with termination in a finite number of steps or after a finite number of terms are computed. Some of the formulations may have removable singularities in the domain of a function. When implementing such formulations, care should be taken in the neighbourhood of singularities to use the appropriate numerical approximations or to isolate the singular points with an open set.

## I.4 Implementations

This International Standard may be implemented in many different ways. Potential implementations include:

a) manual computation without using computers,

b) fixed-purpose hardware, or

c) software executing on general-purpose digital computers ranging from embedded processors to large-scale computer systems.

Given the wide range of possible implementations and the differing requirements of application domains, conformance requirements in this International Standard may be restricted to a sub-set of the domains involved (see Clause 12). (See Annex B for a discussion of computational error, and Clause 14 for specifics on conformance.)

## I.5 Fundamental measure of conformance

There are several conformance criteria that are discussed in Clause 14. One fundamental measure is the numerical difference between the individual data points of an exact or reference set of points, and the corresponding data points generated by a particular implementation. The absolute difference between a data point in the reference data set and the corresponding data point obtained from a particular implementation is referred to as a computational error. The computational error may have units of length, may be angular measures or may be dimensionless, depending on the particular SRF operation being evaluated.

When the reference data are generated, a computational digital accuracy at least as accurate as double precision is assumed, as specified in ISO/IEC/IEEE 60559. This means that the size of the mantissa of a floating-point number is 52 bits, which corresponds to about 15,5 decimal digits of precision (see ISO/IEC/IEEE 60559). Particular implementations may not have to meet this requirement on precision, but developers of the system should understand that use of lower precision arithmetic could increase the computational error when dealing with SRF operations.

## I.6 Error metrics for SRF operations

An error metric is a function that allows data points developed using the exact formulations of Clause 10 to be numerically compared to corresponding data points generated by an implementation. The value of the error metric represents the computational error. Computational errors as defined in this International Standard are absolute errors. These are positive numbers, and may have units of measure associated with them.

Given an exact (or reference) position $(x_0, y_0, z_0)$ in position-space and a computed value $(x, y, z)$ for that position, the error in the computation is given directly in metres by:

$$E = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

where $\Delta x = x - x_0, \Delta y = y - y_0, \Delta z = z - z_0$.

For SRF operations, error metrics are expressed in terms of the coordinate-components of the target SRF. These are obtained from the formulation of $E$ by substituting expressions for $\Delta x, \Delta y, \Delta z$ in terms of the CS coordinate-components of the target SRF.

In the case of a target SRF based on the Euclidean 3D or the Lococentric Euclidean 3D CSs, direct substitution of the (isomorphic) generating functions yields:

$$E = \sqrt{\Delta u^2 + \Delta v^2 + \Delta w^2}$$

where $(\Delta u, \Delta v, \Delta w) = (u, v, w) - (u_0, v_0, w_0)$ is the difference between the exact and computed coordinates.

For a target SRF based on a non-linear CS, and assuming that the error is small, the following approximations for $\Delta x, \Delta y, \Delta z$ apply:

$$\Delta x = \frac{\partial f}{\partial \alpha}\Delta\alpha + \frac{\partial f}{\partial \beta}\Delta\beta + \frac{\partial f}{\partial \gamma}\Delta\gamma$$
$$\Delta y = \frac{\partial g}{\partial \alpha}\Delta\alpha + \frac{\partial g}{\partial \beta}\Delta\beta + \frac{\partial g}{\partial \gamma}\Delta\gamma$$
$$\Delta z = \frac{\partial h}{\partial \alpha}\Delta\alpha + \frac{\partial h}{\partial \beta}\Delta\beta + \frac{\partial h}{\partial \gamma}\Delta\gamma$$

where $(x,y,z) = (f(\alpha,\beta,\gamma), g(\alpha,\beta,\gamma), h(\alpha,\beta,\gamma)) = \boldsymbol{F}(\alpha,\beta,\gamma)$ is the CS generating function, $(\alpha,\beta,\gamma)$ is a computed CS coordinate, $(\alpha_0,\beta_0,\gamma_0)$ is an exact CS coordinate and $(\Delta\alpha,\Delta\beta,\Delta\gamma) = (\alpha,\beta,\gamma) - (\alpha_0,\beta_0,\gamma_0)$.

For a target SRF based on the Equatorial spherical or the Lococentric equatorial spherical CSs, the above approximations yield an expression that may be simplified to:

$$E = \sqrt{(\rho_0 \cos(\theta_0)\,\Delta\lambda)^2 + (\rho_0\Delta\theta)^2 + \Delta\rho^2}$$

where $(\Delta\lambda,\Delta\theta,\Delta\rho) = (\lambda,\theta,\rho) - (\lambda_0,\theta_0,\rho_0)$ is the difference between the exact and computed CS coordinates. The coordinate-component $\rho$ is in metres and the factors $\Delta\lambda$ and $\Delta\theta$ are in (unitless) radians, thus the value $E$ is in metres.

For a target SRF based on the Azimuthal spherical or the Lococentric azimuthal spherical CSs, the above approximations yield an expression that may be simplified to:

$$E = \sqrt{(\rho_0 \cos(\theta_0)\,\Delta\alpha)^2 + \Delta\rho^2 + (\rho_0\Delta\theta)^2}$$

where $(\Delta\alpha,\Delta\rho,\Delta\theta) = (\alpha,\rho,\theta) - (\alpha_0,\rho_0,\theta_0)$ is the difference between the exact and computed CS coordinates. The coordinate-component $\rho$ is in metres and the factors $\Delta\alpha$ and $\Delta\theta$ are in (unitless) radians, thus the value $E$ is in metres.

For a target SRF based on the Geodetic 3D CS, the above approximations yield an expression that may be simplified to:

$$E = \sqrt{\left((\mathcal{N}(\varphi_0) + h_0)\cos(\varphi_0)\,\Delta\lambda\right)^2 + \left((\mathcal{M}(\varphi_0) + h_0)\Delta\varphi\right)^2 + \Delta h^2}$$

where $\mathcal{N}(\varphi)$ and $\mathcal{M}(\varphi)$ are as defined in Table 5.6 and $(\Delta\lambda,\Delta\varphi,\Delta h) = (\lambda,\varphi,h) - (\lambda_0,\varphi_0,h_0)$ is the difference between the exact and computed geodetic coordinates. The ellipsoidal height coordinate-component $h$ is in metres as are the returned values of the functions $\mathcal{M}(\varphi)$ and $\mathcal{N}(\varphi)$. The factors $\Delta\varphi$ and $\Delta\lambda$ are in (unitless) radians, thus the value $E$ is in metres.

For a target SRF based on the Surface geodetic CS, the error expression simplifies to:

$$E = \sqrt{(\mathcal{N}(\varphi_0)\cos(\varphi_0)\,\Delta\lambda)^2 + (\mathcal{M}(\varphi_0)\Delta\varphi)^2}.$$

The Surface geodetic CS formulation for $E$ may be extended to map projection CSs by using the following approximations:

$$\Delta\lambda = \frac{\partial Q_1}{\partial u}\Delta u + \frac{\partial Q_1}{\partial v}\Delta v, \Delta\varphi = \frac{\partial Q_2}{\partial u}\Delta u + \frac{\partial Q_2}{\partial v}\Delta v$$

where $Q_1$ and $Q_2$ are the inverse generating projection functional components, and $u$ and $v$ are the easting and northing coordinate-components.

EXAMPLE For the Equidistant cylindrical CS, which is non-conformal,

$$\Delta\lambda = \tfrac{1}{ak_0}\Delta u + 0, \Delta\varphi = 0 + \tfrac{-1}{R_M(\varphi)}\Delta v$$

 

thus,

$$E = \sqrt{\left(\frac{R_N(\varphi_0)\cos(\varphi_0)\Delta u}{ak_0}\right)^2 + \Delta v^2} = \sqrt{\left(\frac{\Delta u}{k(\lambda_0,\varphi_0)}\right)^2 + \Delta v^2}$$

where $a$ and $k_0$ are CS parameters and $k(\lambda,\varphi)$is the longitudinal point distortion function for the map projection.

For a conformal map projection CS, the error expression simplifies to:

$$E = \frac{\sqrt{\Delta u^2+\Delta v^2}}{k(\lambda_0,\varphi_0)}$$

where $k(\lambda,\varphi)$ is the point distortion function for the map projection CS.

In some SRF operations, such as computing the convergence of the meridian or computing the azimuth, the computed result is a single scalar value $\beta$. In this case, if $\beta_0$ is the exact or reference value and $\beta$ is the computed value, the computational error in radians is:

$$E = |\beta - \beta_0|.$$

In the case of point distortion, the variable is a dimensionless ratio, so that the computational error made in computing $k$ for an exact or reference value $k_0$ is the dimensionless value:

$$E = |k - k_0|.$$

## I.7 Computational error evaluated over test data sets

The previous subclause develops the concept of an error metric that can be used to compare a data point of exact or reference data to the corresponding data point generated by an implementation. It is desirable for the number of test data points to be relatively large, and uniformly distributed over the domain of the operation being evaluated. Legacy implementations of coordinate operations are often verified by using a set of test points that is far too small to properly determine the maximum computational error. If the data set is not large and dense enough, critical points, where the implementation is flawed, may be missed. The description of the size of the test data set and the spatial distribution of values in the set are important considerations, and are operation-dependent. Clause 10 contains descriptions of the domains specific to each operation. Once the domain is specified, the appropriate error metric over the whole set of values can be evaluated and the maximum computational error estimated (see Annex B for dense testing methods). The maximum computational error may be used to determine the level of conformance for a particular implementation.

Methods for determining the computational error over a test data set, and estimating the maximum error can include:

a) calculations performed by hand and supported by a calculating device,

b) calculations performed by hand and supported by a calculating device, and then compared with an existing authoritative data source,

c) construction of a reference implementation in one or more higher-order languages of the exact formulations in Clause 10 to generate a reference data set that can be compared with corresponding points computed by other implementations.

## I.8 Level of conformance

A particular implementation should not be required to meet the standard at the highest level, if this induces unnecessary complexity and cost penalties. In some applications, users may choose to simplify or approximate the formulations to reduce implementation and computational complexity, and, in particular, to reduce computer processing time. In doing so, they are willing to accept some degradation in accuracy for a particular application domain.

EXAMPLE The implementation of a conversion from a celestiodetic SRF to a celestiocentric SRF is tested on the appropriate domain, and the maximum computational error is determined to be less than or equal to 1 mm. This implementation is then said to conform to a 1 mm computational error criterion for that conversion process. Another implementation of the same conversion process, with less stringent requirements, would be said to conform to a 20 cm computational error if the maximum computational error is less than or equal to 20 cm.

The computational accuracy requirement for the default profile, specified in 12.3, is determined by the error bounds and accuracy domain templates contained in the profile specification. Provision is made for registration of profiles having relaxed computational accuracy requirements.



 