7 Reference datums, embeddings, and object reference models

7.1 Introduction

This International Standard specifies ***reference datums*** as geometric primitives in position-space that are used to model aspects of object-space through a process termed reference datum binding. A reference datum binding is an identification of a reference datum in position-space with a corresponding constructed entity in object-space (see 7.2). Reference datums for celestial bodies of interest including Earth are specified in Annex D.

A ***normal embedding*** is a distance-preserving function from position-space to object-space. A normal embedding establishes a model of position-space in object-space by defining an orthonormal frame, termed the embedded frame, in object-space (see 5.2.4). The image of a bound reference datum under a normal embedding may or may not coincide with the constructed entity of the reference datum binding. If they coincide, the reference datum binding and the normal embedding are said to be compatible (see 7.3).

A set of bound reference datums with properly constrained relationships can be selected so as to be compatible with one and only one normal embedding. Such a constrained set of bound reference datums is termed an ***object reference model***. Thus, an object reference model specifies a unique normal embedding. Object reference models generalize the notion of a geodesy datums. Object reference models that use the same set of reference datum primitives and similar binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification (see 7.4).

Object reference models for celestial objects of interest are specified in Annex E. For these celestial objects, one object reference model is designated as the reference model for the object. The transformation from each object reference model to the reference model for the object is termed the reference transformation. A reference transformation is a type of ***similarity transformation*** (see 7.3.2). Similarity transformation templates are defined in 7.3.3 to facilitate the specification of reference transformations. Similarity transformations in general and reference transformations in particular may have time-dependent parameters. Thus, these transformations may be termed time-independent (static) or time-dependent (dynamic). Time-independent reference transformations for celestial object reference models are also specified in Annex E.

Object-specific rules to bind reference datums in a way that is compatible with the binding constraints of an object reference model template are defined in 7.5. These object-specific binding rules are used to provide a uniform method of specifying object reference models for specific dynamically-related celestial bodies.

7.2 Reference datums

7.2.1 Introduction

A *reference datum* (RD) is a geometric primitive in position-space that is used to model an aspect of object-space through a process termed RD binding. In this International Standard, the reference datum concept is defined for 1D, 2D, and 3D position-spaces. In the 2D and 3D cases, this International Standard specifies a small set of reference datums for use in its own specifications. This set is not intended to be exhaustive. Additional RDs may be specified by registration in accordance with Clause 13.

7.2.2 Reference datums

In this International Standard, an RD geometric primitive is expressed in terms of analytic geometry in position-space. RDs are designed to correspond to constructed entities of similar geometric type in an object-space through a process termed RD binding (see 7.2.5). These geometric types are limited to a point, a directed curve, or an oriented surface. The analytic form of the position-space representation and its corresponding object-space geometric representation are described by category and position-space dimension in Table 7.1. An RD of a given category is specified by the parameters and/or the analytic expression of its position-space representation.

Table 7.1 — RD categories

RD category	Position-space representation			Object-space representation
	1D	2D	3D	
Point	$[a]$ real a	$[a, b]$ real a, b	$[a, b, c]$ real a, b, c	a point in the object-space
Directed curve		$\boldsymbol{p} = \boldsymbol{F}(t)$, $\boldsymbol{F}$ is smooth, $\mathbb{R}^2$ valued with domain $D \subseteq \mathbb{R}^1$. Direction at $\boldsymbol{p_0} = \boldsymbol{F}(t_0)$ is $\boldsymbol{n} = \frac{d\boldsymbol{F}}{dt}(t_0)$.	$\boldsymbol{p} = \boldsymbol{F}(t)$, $\boldsymbol{F}$ is smooth, $\mathbb{R}^3$ valued with domain $D \subseteq \mathbb{R}^1$. Direction at $\boldsymbol{p_0} = \boldsymbol{F}(t_0)$ is $\boldsymbol{n} = \frac{d\boldsymbol{F}}{dt}(t_0)$.	a curve in the object-space with a designation of direction along the curve
Oriented surface			Implicit definition: $f(\boldsymbol{p}) = 0$. f is smooth, $\mathbb{R}^1$ valued for $\boldsymbol{p}$ in position-space. Positive side of surface (orientation): $f(\boldsymbol{p}) > 0$.	a surface in the object-space with a designation of one side as positive

This International Standard specifies 2D and 3D RDs by RD category in Table 7.4 through Table 7.8. The specification elements of those tables are defined in Table 7.2. 3D RDs based on ellipsoids are described in 7.2.3 and 7.2.4 and specified in Annex D with specification elements defined in Table 7.9. Table 7.3 is a directory of RD specification tables or, in the case of 3D RDs based on ellipsoids, RD specification directories.

Table 7.2 — RD specification elements

Element	Definition
RD label	The label for the RD (see 13.2.2).
RD code	The code for the RD (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Description	A description of the RD including any common name for the concept.
Position-space representation	The analytic formulation of the RD in position-space.

Table 7.3 — RD specification directory

Position-space dimension	RD category	Table number
2D	point	Table 7.4
3D	point	Table 7.5
2D	directed curve	Table 7.6
3D	directed curve	Table 7.7
3D	oriented surface	Table 7.8 and Table 7.10

Table 7.4 — 2D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_2D	1	Origin in 2D	[0,0]
X_UNIT_POINT_2D	2	x-axis unit point in 2D	[1,0]
Y_UNIT_POINT_2D	3	y-axis unit point in 2D	[0,1]

Table 7.5 — 3D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_3D	4	Origin in 3D	[0,0,0]
X_UNIT_POINT_3D	5	x-axis unit point in 3D	[1,0,0]
Y_UNIT_POINT_3D	6	y-axis unit point in 3D	[0,1,0]
Z_UNIT_POINT_3D	7	z-axis unit point in 3D	[0,0,1]

Table 7.6 — 2D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_2D	8	x-axis in 2D	$\boldsymbol{F}(t) \equiv [t, 0]$
Y_AXIS_2D	9	y-axis in 2D	$\boldsymbol{F}(t) \equiv [0, t]$

Table 7.7 — 3D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_3D	10	x-axis in 3D	$\boldsymbol{F}(t) \equiv [t, 0,0]$
Y_AXIS_3D	11	y-axis in 3D	$\boldsymbol{F}(t) \equiv [0, t, 0]$
Z_AXIS_3D	12	z-axis in 3D	$\boldsymbol{F}(t) \equiv [0,0, t]$

Table 7.8 — 3D RDs of category oriented surface

RD label	RD code	Description	Position-space representation
XY_PLANE_3D	13	xy-plane	$0 = f(x, y, z) \equiv z$
XZ_PLANE_3D	14	xz-plane	$0 = f(x, y, z) \equiv y$
YZ_PLANE_3D	15	yz-plane	$0 = f(x, y, z) \equiv x$

7.2.3 Ellipsoidal RDs

The RDs specified in this International Standard include RDs based on oblate ellipsoids, prolate ellipsoids, and tri-axial ellipsoids. These RDs are 3D and of RD category oriented surface. These RDs are specified based upon certain geometrically defined parameters. The position-space representations of oblate and prolate ellipsoid RDs are expressed in the form:

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0 \text{ where } a \neq 0 \neq b. \quad (7.1)$$

When $a \geq b$, an RD of this form is an *oblate ellipsoid RD* with major semi-axis a and minor semi-axis b as illustrated in Figure 7.1.

Spheres shall be considered as a special case of oblate ellipsoids, where appropriate. If $a = b$, an oblate ellipsoid RD may be termed a *sphere RD*. In this case, the value $r = a = b$ is the radius of the sphere RD.

NOTE In general usage, spheres are a limiting case of oblate, prolate, and tri-axial ellipsoids.

When $a < b$, an RD of this form is a *prolate ellipsoid RD* with major semi-axis b and minor semi-axis a, as illustrated in Figure 7.1.

Instead of specifying the parameters of an oblate ellipsoid RD as the major semi-axis a and the minor semi-axis b, it is both equivalent and sometimes convenient to use the major semi-axis a and the flattening f as defined in Equation 7.2. The minor semi-axis b may be expressed in terms of the major semi-axis a and the flattening f as in Equation 7.3. The flattening of a sphere RD is zero ($f = 0$).

flattening definition: $f \equiv \frac{a-b}{a}$ (7.2)

Minor semi-axis relationship: $b = a - af$ (7.3)

The position-space representation of a tri-axial ellipsoid RD is expressed in the form:

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \quad (7.4)$$

The semi-axes a, b, and c shall be positive non-zero and $a \neq b \neq c \neq a$.

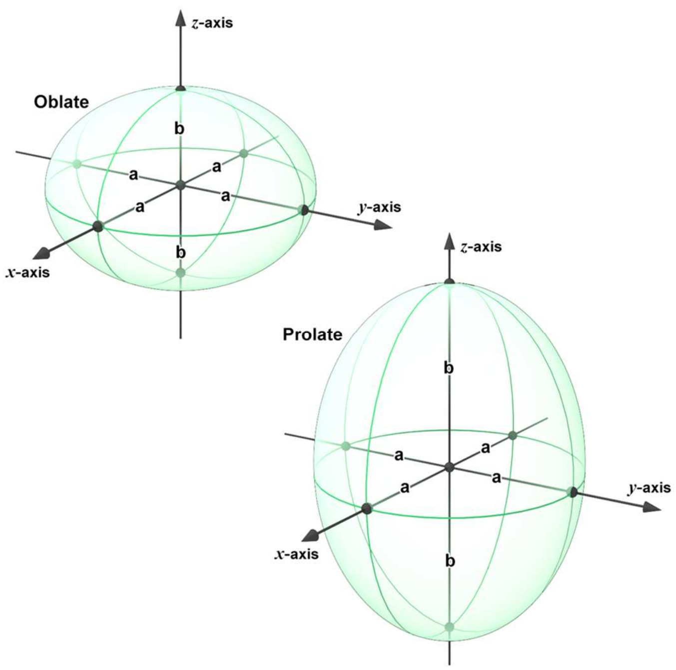


Figure 7.1 — Oblate and prolate ellipsoids

7.2.4 RDs associated with physical objects

In the case of ellipsoid RDs intended for modelling physical objects of interest, published parameter values for these RDs are used. The specification of these RDs includes the published ellipsoid parameters and the identification of the associated physical object. The specification elements for physical object RDs are defined in Table 7.9.

Table 7.9 — Physical object RD specification elements

<table>
<tr><th>Element</th><th colspan="2">Specification</th></tr>
<tr><td>RD label</td><td colspan="2">The label for the RD (see 13.2.2).</td></tr>
<tr><td>RD code</td><td colspan="2">The code for the RD (see 13.2.3).</td></tr>
<tr><td>Description</td><td colspan="2">The description including the name of the physical object as published or as commonly known.</td></tr>
<tr><td rowspan="5">Parameters</td><td>Oblate ellipsoid case</td><td>Major semi-axis, a
Flattening, f</td></tr>
<tr><td>Sphere case</td><td>Radius, r</td></tr>
<tr><td>Prolate ellipsoid case</td><td>Minor semi-axis, a
Major semi-axis, b</td></tr>
<tr><td>Tri-axial ellipsoid case</td><td>x-semi-axis, a
y-semi-axis, b
z-semi-axis, c</td></tr>
<tr><td colspan="2">RD parameters shall be specified by value or by reference (see 13.2.5).
If by value, the value(s) shall be followed by an error estimate expressed in one of the following forms:
a) error estimate: unknown
b) error estimate: assumed precise
c) error estimate (1σ): <parameter name>:<error value>
d) error interval: <parameter name> ± <error value>
EXAMPLE error estimate (1σ): a: 1 250, f^{-1}: 0,25.
If by reference, this specification element shall express the value(s) and error estimate(s) using the terminology found in the reference. These terms shall be enclosed in brackets ({}). Any parameter value that is not specified in the citation(s) shall be specified as in the “by value” case. An error estimate for b or for f^{-1} may be substituted in place of an error estimate for f.</td></tr>
<tr><td>Date</td><td colspan="2">The date the RD parameters were specified or published.</td></tr>
<tr><td>References</td><td colspan="2">The references (see 13.2.5).</td></tr>
</table>

The RDs associated with physical objects are specified in Annex D. Table 7.10 is a directory of these RDs organized by type of ellipsoid. The semi-axis and radius parameters are unitless in position-space but are bound to metre lengths when the RD is identified with the corresponding physical object-space constructed entity.

Table 7.10 — Physical RD specification table locations

Type of ellipsoid	RD table
Oblate ellipsoid	Table D.2
Sphere	Table D.3
Prolate ellipsoid	Table D.4
Tri-axial ellipsoid	Table D.5

7.2.5 RD binding

An RD is *bound* when the RD in position-space is identified with a corresponding constructed entity in object-space. In this context, a "constructed entity" is defined to mean an intrinsic, artificial, measured, or conceptual entity in object-space that is uniquely identifiable within the user's application domain. The term "corresponding" in this context means that each RD is bound to a constructed entity of the same geometric object type. That is, position-space points are bound to identified points in object-space, position-space directed lines to constructed lines or line segments in object-space, position-space directed curves to constructed curves or curve segments in object-space, position-space oriented planes to constructed planes or partial planes in object-space, and position-space oriented surfaces to constructed surfaces or partial surfaces in object-space.

When a curve or surface RD is bound, the radii of curvature on the corresponding constructed entity in object-space shall correspond to the radii of curvature in position-space. In this International Standard, in the case of physical objects, one unit in position-space corresponds to one metre in object-space. In the case of abstract objects, one unit in position-space corresponds to the designated length scale unit in the abstract object-space. In particular, the semi-axes of an ellipsoid RD shall correspond to the semi-axes of the constructed ellipsoid to which it is bound.

If the constructed entity of an RD binding is fixed with respect to object-space, then the RD binding shall be termed an *object-fixed RD binding.* This definition assumes that the constructed entity does not move over time by an amount significant for the accuracy and time scale of an application.

Figure 7.2 illustrates two distinct bindings of a point RD. On the left, it is bound to a specific point in the abstract object-space of a CAD/CAM model. On the right, it is bound to a point in physical object-space that is on an object that has been manufactured from that CAD model.

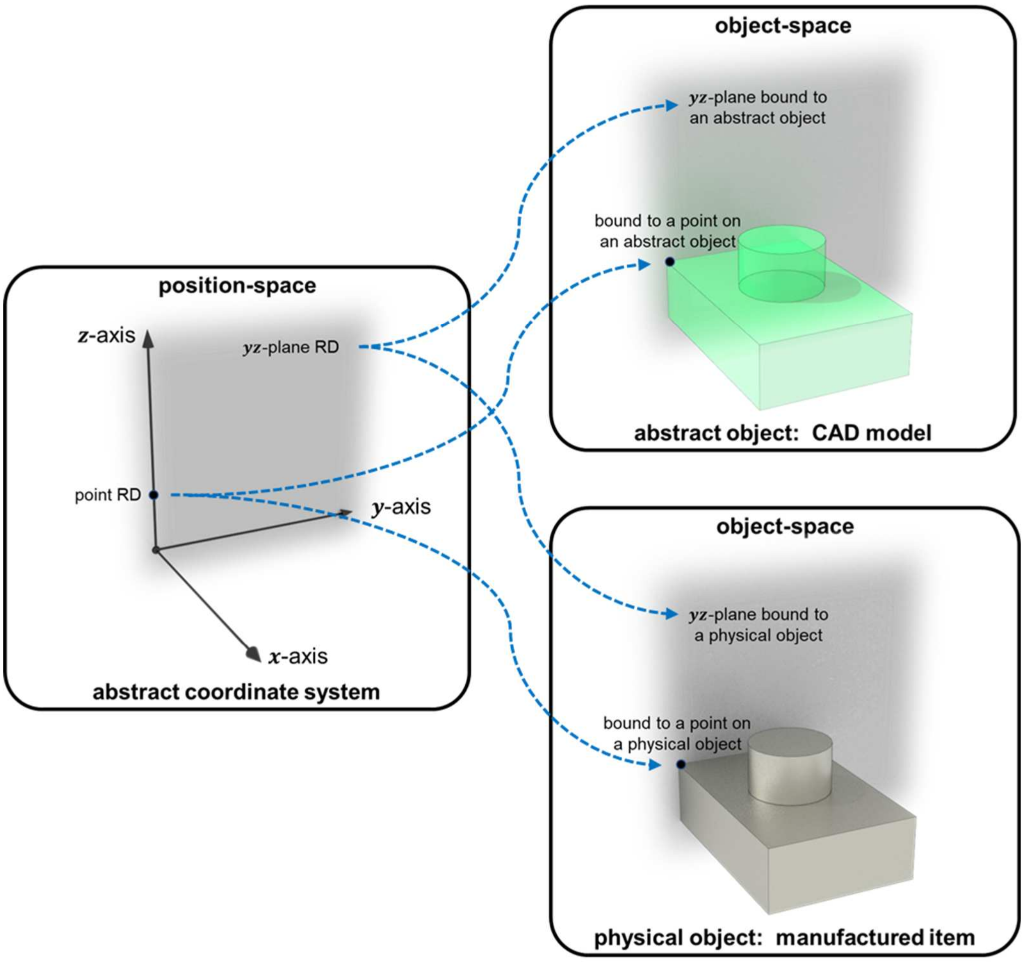


Figure 7.2 — Two RDs bound to an abstract object and to a physical object

In some application domains, bound reference datums are used to model a significant aspect of the problem domain. In geodesy, oblate ellipsoids are used to model the figure of the Earth or a subset thereof.

EXAMPLE 1 An ellipsoid reference datum with major semi-axis a and minor semi-axis b is the surface implicitly defined by:

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0$$

and is illustrated in Figure 7.3.

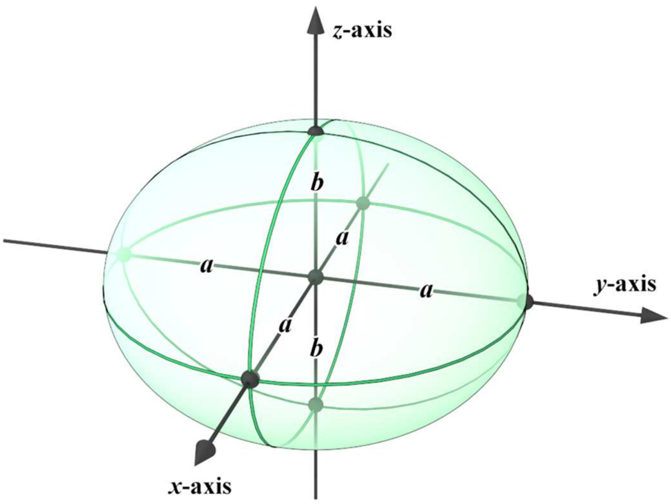


Figure 7.3 — An ellipsoid reference datum

EXAMPLE 2 Semi-axis values a and b, $a \geq b$, are selected to specify an oblate ellipsoid reference datum. The following steps (see Figure 7.4) illustrate one way to bind an ellipsoid reference datum specified by semi-axis values a and b to a conceptual ellipsoid that represents the figure of the Earth in a region as approximated by a geodetic survey control network:

a) A point on the surface of the reference datum is specified. This point has a computable geodetic latitude φ.

b) The specified position-space point is identified with a specific point in object-space.

c) The direction of the oblate ellipsoid rotational axis is constructed in object-space.

d) The direction of the outward surface normal at the point is constructed in object-space so that the angle it makes with respect to the oblate ellipsoid rotational axis direction is $(\pi/2 - \varphi)$.

This binding requires the specification of a point and two directions in object-space.

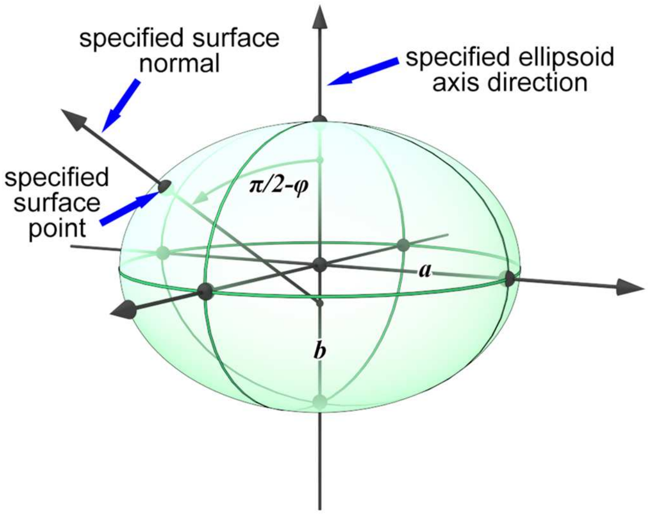


Figure 7.4 — A reference datum binding

7.3 Normal embeddings and similarity transformations

7.3.1 Normal embeddings

A normal embedding is a distance-preserving function from position-space to object-space (see 5.2.5). A normal embedding functionally maps the canonical origin and basis vectors of position-space to an orthonormal frame in object-space. The normal embedding thereby forms a vector space isomorphism between positions space and the vector-space generated by the orthonormal frame in object-space, thus establishing a model of position-space in object-space.

7.3.2 Similarity transformations

Many normal embeddings can be specified for a 3D object-space. Two embedded frames determined by two such embeddings are denoted by $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$, respectively. A point $\boldsymbol{p}$ in object-space will have coordinates $[x_1, y_1, z_1]_{\boldsymbol{E1}}$ in terms of embedded frame $\boldsymbol{E_1}$ and $[x_2, y_2, z_2]_{\boldsymbol{E2}}$ in terms of embedded frame $\boldsymbol{E_2}$. The origin of $\boldsymbol{E_2}$ may be displaced with respect to the origin of $\boldsymbol{E_1}$, and the corresponding axes may point in differing directions (see Figure 7.5). For 2D object-spaces, similar relationships hold.

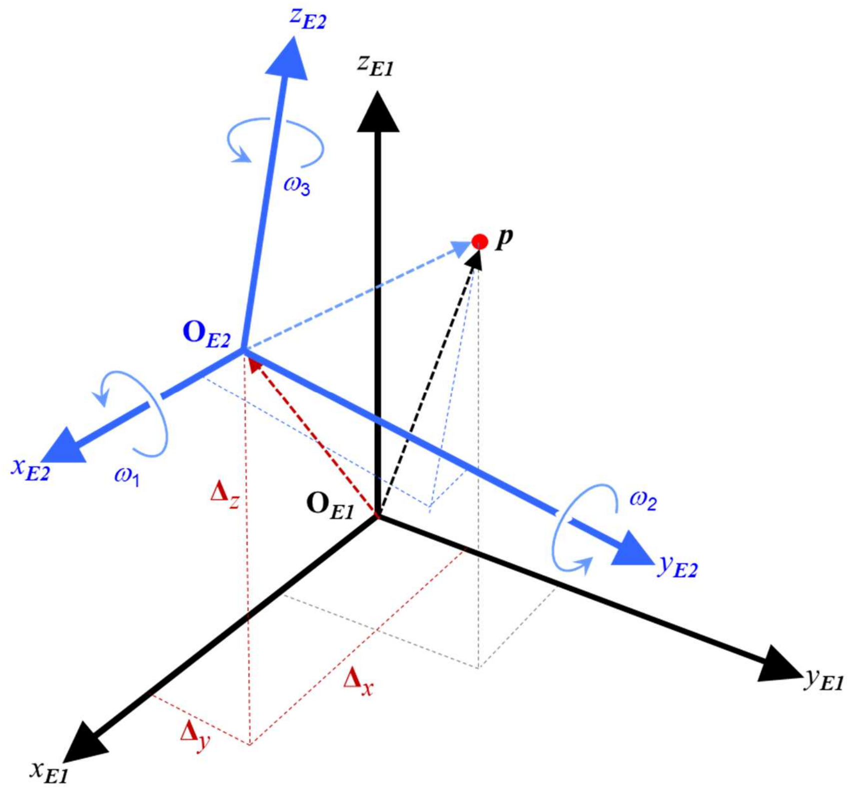


Figure 7.5 — 3D normal embedding relationships

A similarity transformation is defined as a transformation in object-space that re-expresses a position that is expressed in terms of one embedded frame in terms of another embedded frame. In its most general form, a similarity transformation performs a translation, a rotation, and a scaling operation. For computations involving abstract object-spaces, the scale adjustment is needed to account for differing length scales. In the case of physical object-spaces, scale factors, commonly close to 1,0 in value, are often required to adjust for spatial distortions in empirically estimated data. Scaling is addressed in 7.4.6. Similarity transformations play a central role in operations on object reference models (see 10.3).

The general form of a similarity transformation, denoted $\boldsymbol{H}_{E1 \leftarrow E2}$, that re-expresses a position expressed in frame $\boldsymbol{E}_1$ in terms of frame $\boldsymbol{E}_2$, is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E1} = \boldsymbol{H}_{E1 \leftarrow E2} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E2}$$

Similarity transformations can be expressed as a generalization of the change of basis operation $\boldsymbol{\Omega}_{E1 \leftarrow E2}$ defined in 6.2.2, adding translation and scaling. The translation component of the similarity transformation is the vector from the origin of frame $\boldsymbol{E}_1$ to the origin of frame $\boldsymbol{E}_2$, denoted by $\overrightarrow{O_{E1} O_{E2}}$, which is the origin of frame $\boldsymbol{E}_2$ expressed in terms of frame $\boldsymbol{E}_1$. For consistency with the other components of the similarity transformation, the vector $\overrightarrow{O_{E1} O_{E2}}$ can be denoted by $\vec{\boldsymbol{\Delta}}_{E1 \leftarrow E2}$. In a similar manner, the scaling component of the similarity transformation can be denoted by $\boldsymbol{\sigma}_{E1 \leftarrow E2}$. The similarity transformation is given in terms of these components by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E1} = \vec{\boldsymbol{\Delta}}_{E1 \leftarrow E2} + \boldsymbol{\sigma}_{E1 \leftarrow E2} \boldsymbol{\Omega}_{E1 \leftarrow E2} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E2}$$

where:
$\vec{\boldsymbol{\Delta}}_{E1 \leftarrow E2} = [\Delta x \quad \Delta y \quad \Delta z]^{\mathrm{T}}$ is the displacement vector from the origin of $\boldsymbol{E}_1$ to the origin of $\boldsymbol{E}_2$,
$\boldsymbol{\sigma}_{E1 \leftarrow E2}$ is the scale factor of $\boldsymbol{E}_1$ with respect to $\boldsymbol{E}_2$, and
$\boldsymbol{\Omega}_{E1 \leftarrow E2}$ is the orientation (i.e., angular displacement) of the basis vectors of $\boldsymbol{E}_1$ with respect to the basis vectors of $\boldsymbol{E}_2$,

Similarity transformations can also be expressed in terms of an equivalent rotation operation (see 6.2.5), along with scaling and displacement, as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E1} = \vec{\boldsymbol{\Delta}}_{E1 \leftarrow E2} + \boldsymbol{\sigma}_{E1 \leftarrow E2} \boldsymbol{R}_{E1 \rightarrow E2} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E2}$$

where:
$\boldsymbol{R}_{E1 \rightarrow E2}$ rotates the basis vectors of $\boldsymbol{E}_1$ to align with the basis vectors of $\boldsymbol{E}_2$,

This rotation operation can be expressed in any of the forms defined in 6.6, including axis-angle, matrix, Euler angle conventions, or quaternions. A sequence of three principal axis rotation operations is commonly used.

Similarity transformations are typically expressed in what is commonly termed the Position Vector Rotation (PVR) convention. In the PVR convention, the principal axis rotations are as given in Table 6.1:

$$\boldsymbol{R}_X\langle\omega_1\rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & -\sin(\omega_1) \\ 0 & \sin(\omega_1) & \cos(\omega_1) \end{bmatrix}$$
$$\boldsymbol{R}_Y\langle\omega_2\rangle = \begin{bmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix}$$
$$\boldsymbol{R}_Z\langle\omega_3\rangle = \begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The principal rotation order is expressed left-to-right using the space-fixed equivalent of body-fixed convention (see 6.4.2.4), and the direction of positive rotation angles is defined in accordance with the right-hand rule (see 5.2.3).

The generalized Helmert 7-parameter transformation used in geodesy is expressed in the PVR convention (see Table 7.24) using rotation angles $\omega_1, \omega_2, \omega_3$ as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E1} = \vec{\boldsymbol{\Delta}}_{\boldsymbol{E1 \leftarrow E2}} + \boldsymbol{\sigma}_{\boldsymbol{E1 \leftarrow E2}} \boldsymbol{R}_X \langle \omega_1 \rangle \circ \boldsymbol{R}_Y \langle \omega_2 \rangle \circ \boldsymbol{R}_Z \langle \omega_3 \rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E2}$$

where:

ω_1: the $\boldsymbol{E_2}\,\boldsymbol{x}$-axis angle of rotation in the PVR convention,

ω_2: the $\boldsymbol{E_2}\,\boldsymbol{y}$-axis angle of rotation in the PVR convention,

ω_3: the $\boldsymbol{E_2}\,\boldsymbol{z}$-axis angle of rotation in the PVR convention, and where

$\boldsymbol{R}_X \langle \omega_1 \rangle \circ \boldsymbol{R}_Y \langle \omega_2 \rangle \circ \boldsymbol{R}_Z \langle \omega_3 \rangle =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & -\sin(\omega_1) \\ 0 & \sin(\omega_1) & \cos(\omega_1) \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & -\cos(\omega_2)\sin(\omega_3) & \sin(\omega_2) \\ \cos(\omega_1)\sin(\omega_3) + \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) & \cos(\omega_1)\cos(\omega_3) - \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) & -\sin(\omega_1)\cos(\omega_2) \\ \sin(\omega_1)\sin(\omega_3) - \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) & \sin(\omega_1)\cos(\omega_3) + \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$$

However, in some sources, similarity transformations are also expressed using a different convention, which is commonly termed the Coordinate Frame Rotation (CFR) convention. When applied properly, the two conventions, given the same input, will produce the same result. In the CFR convention, the principal rotation matrices are inverted (i.e., transposed), are applied in reverse order, and the direction of positive rotation angle values is negated with respect to the right-hand rule.

$$\boldsymbol{R}_{\boldsymbol{X}}^{-\boldsymbol{1}} \langle \omega_1 \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & \sin(\omega_1) \\ 0 & -\sin(\omega_1) & \cos(\omega_1) \end{bmatrix}$$

$$\boldsymbol{R}_{\boldsymbol{Y}}^{-\boldsymbol{1}} \langle \omega_2 \rangle = \begin{bmatrix} \cos(\omega_2) & 0 & -\sin(\omega_2) \\ 0 & 1 & 0 \\ \sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix}$$

$$\boldsymbol{R}_{\boldsymbol{Z}}^{-\boldsymbol{1}} \langle \omega_3 \rangle = \begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Since $\sin(-\omega) = -\sin(\omega)$, the effects of 1) using the inverse rotation matrices in reversed order, and 2) negating the direction of the rotation angles, cancel each other. While the CFR convention produces correct results, these multiple reversals can be confusing, and can become a common source of errors.

The generalized Helmert 7-parameter transformation used in geodesy (see Table 7.25) is expressed in the CFR convention using rotation angles $\omega_1, \omega_2, \omega_3$ as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E1} = \vec{\boldsymbol{\Delta}}_{\boldsymbol{E1 \leftarrow E2}} + \boldsymbol{\sigma}_{\boldsymbol{E1 \leftarrow E2}} \boldsymbol{R}_{\boldsymbol{Z}}^{-\boldsymbol{1}} \langle \omega_3 \rangle \circ \boldsymbol{R}_{\boldsymbol{Y}}^{-\boldsymbol{1}} \langle \omega_2 \rangle \circ \boldsymbol{R}_{\boldsymbol{X}}^{-\boldsymbol{1}} \langle \omega_1 \rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{E2}$$

where:

ω_1: the $\boldsymbol{E_2}\,\boldsymbol{x}$-axis angle of rotation in the CFR convention,

ω_2: the $\boldsymbol{E_2}\,\boldsymbol{y}$-axis angle of rotation in the CFR convention,

ω_3: the $\boldsymbol{E_2}\,\boldsymbol{z}$-axis angle of rotation in the CFR convention, and where

$\boldsymbol{R}_{\boldsymbol{Z}}^{-\boldsymbol{1}} \langle \omega_3 \rangle \circ \boldsymbol{R}_{\boldsymbol{Y}}^{-\boldsymbol{1}} \langle \omega_2 \rangle \circ \boldsymbol{R}_{\boldsymbol{X}}^{-\boldsymbol{1}} \langle \omega_1 \rangle =$

$$\begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & -\sin(\omega_2) \\ 0 & 1 & 0 \\ \sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & \sin(\omega_1) \\ 0 & -\sin(\omega_1) & \cos(\omega_1) \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & \cos(\omega_1)\sin(\omega_3) + \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) & \sin(\omega_1)\sin(\omega_3) - \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) \\ -\cos(\omega_2)\sin(\omega_3) & \cos(\omega_1)\cos(\omega_3) - \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) & \sin(\omega_1)\cos(\omega_3) + \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) \\ \sin(\omega_2) & -\sin(\omega_1)\cos(\omega_2) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$$

7.3.3 Similarity transformation templates

In both 2D and 3D position-space, there are several ways to represent a similarity transformation. The particular representation of these operations may depend on the domain of a user application and on the types of physical and data models used to determine the appropriate similarity transformation. The number of parameters used in various representations, specified below, varies from zero parameters for the identity transformation to 13 parameters for one of the general matrix formulations.

A *similarity transformation template* (STT) is a mechanism for parametrically specifying a similarity transformation. Each STT is given a label and a code. An instance of a similarity transformation is specified by providing a label or code of an STT and values for all of the parameters listed in that STT.

The use of STTs adds flexibility and simplicity to the process of specifying a reference transformation (see 7.4.6) or other similarity transformation. By using the appropriate STT, a reference transformation can be specified with the original source data values for STT parameters. The elements of an STT specification are defined in Table 7.11. The STT specification elements include STT formulation and STT inverse formulation elements. These mathematical formulations define the similarity transformations in terms of the STT parameter values.

In the case of a similarity transformation that serves as a reference transformation for an object reference model, the constraint that one unit in object-space must have length one metre should disallow non-unit scaling. However, many local object reference model for Earth have reference transformations that are determined as a best fit to empirical data for which a very small scale adjustment provides an additional degree of freedom to reduce the residual error of the fit [RAPP2]. These small-scale adjustments are permitted when specifying a reference transformation for a local object reference model.

An STT instance is realized by providing values for the parameters of an STT. If one or more of the parameters are evaluated as functions of time, the realized similarity transformation is itself a function of time. Examples of such dynamic similarity transformations are presented in 7.5. Holding the time variable fixed to a specific value allows the dynamic case to be treated as a static case.

Table 7.11 — STT specification elements

Element	Definition
STT label	The label for the STT (see 13.2.2).
STT code	The code for the STT (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Name(s)	The name or names given to this form of similarity transformation.
Description	A short description.
Dimension	The domain dimension indicated as "2D" or "3D".
STT parameters	Parameters shall be listed in a specified order using the following format: *symbol* : *name or description* in *unit of measure* or *symbol* : *name or description* (unitless) or none
STT constraints	Parametric constraints, if any, or "none".

Element	Definition
STT formulation	An equation for the similarity transformation in terms of the STT parameters and the source position $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_S$ and target $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_T$ position.
STT inverse formulation	An equation for the inverse similarity transformation in terms of the STT parameters and the source and target positions.
Note(s)	Optional notes.
Reference(s)	Bibliographic reference(s) (see 13.2.5), or "none" if defined in this International Standard.

This International Standard provides a collection of STT specifications as identified in Table 7.12. Standardized STTs are specified in Tables 7.13 through 7.28. Additional STTs may be specified by registration in accordance with Clause 13.

Table 7.12 — Similarity transformation template directory

Similarity transformation template name	Table number
Identity transformation 3D	Table 7.13
Identity transformation 2D	Table 7.14
Translation transformation 3D	Table 7.15
Translation transformation 2D	Table 7.16
Simplified Helmert (Bursa-Wolf) 7-parameter transformation (PVR convention)	Table 7.17
Simplified Helmert (Bursa-Wolf) 7-parameter transformation (CFR convention)	Table 7.18
Molodensky-Badekas (Bursa-Wolf) 7+3-parameter transformation (CFR convention)	Table 7.19
General rotate-scale-translate transformation 3D	Table 7.20
General rotate-scale-translate transformation 2D	Table 7.21
Homogeneous matrix 4x4 transformation 3D	Table 7.22
Homogeneous matrix 3x3 transformation 2D	Table 7.23
Generalized Helmert rotate-scale-translate transformation (PVR convention)	Table 7.24
Generalized Helmert rotate-scale-translate transformation (CFR convention)	Table 7.25
Tait-Bryan z-y-x rotate-translate transformation	Table 7.26
Non-Greenwich prime meridian z rotate-translate transformation	Table 7.27
Geomagnetic z-y rotate transformation	Table 7.28

7.3.3.1 Identity transformation 3D

Table 7.13 — Identity 3D STT

Element	Definition
STT label	IDENTITY
STT code	1
Name(s)	Identity transformation 3D
Description	Identity transformation in three-dimensional object-space.

Element	Definition
Dimension	3D
STT parameters	None
STT constraints	None
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \boldsymbol{I}_{\mathrm{T}\leftarrow\mathrm{S}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$, where $\boldsymbol{I}_{\mathrm{T}\leftarrow\mathrm{S}}$ is the identify matrix
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \boldsymbol{I}_{\mathrm{S}\leftarrow\mathrm{T}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}}$, where $\boldsymbol{I}_{\mathrm{S}\leftarrow\mathrm{T}}$ is the identify matrix
Note(s)	This STT is used for the object reference RT and other identity RTs.
Reference(s)	none

7.3.3.2 Identity transformation 2D

Table 7.14 — Identity 2D STT

Element	Definition
STT label	IDENTITY_2D
STT code	2
Name(s)	Identity transformation 2D
Description	Identity transformation in two-dimensional object-space.
Dimension	2D
STT parameters	None
STT constraints	None
STT formulation	$\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{T}} = \boldsymbol{I}_{\mathrm{T}\leftarrow\mathrm{S}} \begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{S}}$, where $\boldsymbol{I}_{\mathrm{T}\leftarrow\mathrm{S}}$ is the identify matrix
STT inverse formulation	$\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{S}} = \boldsymbol{I}_{\mathrm{S}\leftarrow\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{T}}$, where $\boldsymbol{I}_{\mathrm{S}\leftarrow\mathrm{T}}$ is the identify matrix
Note(s)	This STT is used for the object reference RT and other identity RTs.
Reference(s)	None

7.3.3.3 Translation transformation 3D

Table 7.15 — Translation 3D STT

Element	Definition
STT label	TRANSLATE
STT code	3
Name(s)	Translation transformation 3D
Description	Translated origin in three-dimensional object-space.
Dimension	3D

STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres Δz: the z-component of the displacement from target origin to source origin in metres.
STT constraints	None
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}} + \boldsymbol{I}_{\mathrm{T} \leftarrow \mathrm{S}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$, where $\boldsymbol{I}_{\mathrm{T} \leftarrow \mathrm{S}}$ is the identify matrix
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \boldsymbol{I}_{\mathrm{S} \leftarrow \mathrm{T}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}}$, where $\boldsymbol{I}_{\mathrm{S} \leftarrow \mathrm{T}}$ is the identify matrix
Note(s)	This is a common RT form for local ERMs.
Reference(s)	NGA36]

7.3.3.4 Translation transformation 2D

Table 7.16 — Translation 2D STT

Element	Definition
STT label	TRANSLATE_2D
STT code	4
Name(s)	Translation transformation 2D
Description	Translated origin in two-dimensional object-space.
Dimension	2D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres.
STT constraints	None
STT formulation	$\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}} + \boldsymbol{I}_{\mathrm{T} \leftarrow \mathrm{S}} \begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{S}}$, where $\boldsymbol{I}_{\mathrm{T} \leftarrow \mathrm{S}}$ is the identify matrix
STT inverse formulation	$\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{S}} = \boldsymbol{I}_{\mathrm{S} \leftarrow \mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}}$, where $\boldsymbol{I}_{\mathrm{S} \leftarrow \mathrm{T}}$ is the identify matrix
Note(s)	None
Reference(s)	None

7.3.3.5 Simplified Helmert transformation (PVR convention)

Table 7.17 — Simplified Helmert STT (PVR convention)

Element	Definition
STT label	PV_7_PARAMETER
STT code	5
Name(s)	Simplified Helmert (Bursa-Wolf) 7-parameter transformation (PVR convention)
Description	Helmert transformation using the Bursa-Wolf small angle approximation of the rotation matrix with angles in the PVR convention.
Dimension	3D

Element	Definition
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. ω_1: the source $\boldsymbol{x}$-axis angle of rotation in radians in the PVR convention. ω_2: the source $\boldsymbol{y}$-axis angle of rotation in radians in the PVR convention. ω_3: the source $\boldsymbol{z}$-axis angle of rotation in radians in the PVR convention. Δs: the scale difference from unity (unitless).
STT constraints	1) ω_1, ω_2 and, ω_3 are small rotations (magnitude less than $2\text{x}10^{-4}$ radians) in the PVR convention, 2) Δs is a small adjustment of scale ($\lvert\Delta s\rvert < 10^{-5}$).
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T \leftarrow S}} + (1 + \Delta s) \begin{bmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = (1 - \Delta s) \begin{bmatrix} 1 & \omega_3 & -\omega_2 \\ -\omega_3 & 1 & \omega_1 \\ \omega_2 & -\omega_1 & 1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T \leftarrow S}} \right)$
Note(s)	1) This transformation is widely used in geodesy with rotation angle values specified in arc second or milliarcsecond (mas) units. 2) The matrix in the formulation is the Bursa-Wolf small angle approximation of a rotation matrix. When multiplied with its transverse, the matrix product approximates the identity matrix with a matrix element-wise absolute error of 10^{-8} or less. The factor $(1 - \Delta s)$ approximates $1/(1 + \Delta s)$ with absolute error $<10^{-10}$. 3) The approximation uses the results that $\lvert\omega - \sin(\omega)\rvert < \omega^3/6$ and $\lvert 1 - \cos(\omega)\rvert < \omega^2/2$ when ω is expressed in radians. When $\omega = 1''$, the error multiplied by the radius of the Earth is less than 75 µm. 4) An alternative form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T \leftarrow S}} + \begin{bmatrix} 1 + \Delta s & -\omega_3 & \omega_2 \\ \omega_3 & 1 + \Delta s & -\omega_1 \\ -\omega_2 & \omega_1 & 1 + \Delta s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}.$ The difference between the two forms is negligible when the parameter constraints are met (10^{-8} magnitude error or less).
Reference(s)	[RAPP2], [GN72]

7.3.3.6 Simplified Helmert transformation (CFR convention)

Table 7.18 — Simplified Helmert STT (CFR convention)

Element	Definition
STT label	CF_7_PARAMETER
STT code	6
Name(s)	Simplified Helmert (Bursa-Wolf) 7-parameter transformation (CFR convention)
Description	Helmert transformation using the Bursa-Wolf small angle approximation of the rotation matrix, with angles in the CFR convention.
Dimension	3D

Element	Definition
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. ω_1: the source $\boldsymbol{x}$-axis angle of rotation in radians in the CFR convention. ω_2: the source $\boldsymbol{y}$-axis angle of rotation in radians in the CFR convention. ω_3: the source $\boldsymbol{z}$-axis angle of rotation in radians in the CFR convention. Δs: the scale difference from unity (unitless).
STT constraints	1) ω_1, ω_2 and, ω_3 are small rotations (magnitude less than 2x10^{-4} radians) in the CFR convention, 2) Δs is a small adjustment of scale ($\lvert\Delta s\rvert < 10^{-5}$).
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}} + (1+\Delta s)\begin{bmatrix} 1 & \omega_3 & -\omega_2 \\ -\omega_3 & 1 & \omega_1 \\ \omega_2 & -\omega_1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = (1-\Delta s)\begin{bmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}}\right)$
Note(s)	1) This is a traditional transformation used in geodesy with rotation angle values often specified in arc second or milliarcsecond (mas) units. 2) The factor $(1-\Delta s)$ approximates $1/(1+\Delta s)$ with absolute error <10^{-10}. The matrix in the formulation is the Bursa-Wolf small angle approximation of a rotation matrix. When multiplied with its transverse, the matrix product approximates the identity matrix with a matrix element-wise absolute error of 10^{-8} or less. The approximation uses the results that $\lvert\omega - \sin(\omega)\rvert < \omega^3/6$ and $\lvert 1-\cos(\omega)\rvert < \omega^2/2$ when ω is expressed in radians. When $\omega = 1$", the error multiplied by the radius of the Earth is less than 75 µm. 3) An alternative form of the STT formulation is $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T} \leftarrow \mathrm{S}} + \begin{bmatrix} 1+\Delta s & \omega_3 & -\omega_2 \\ -\omega_3 & 1+\Delta s & \omega_1 \\ \omega_2 & -\omega_1 & 1+\Delta s \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}.$ The difference between the two forms is negligible when the parameter constraints are met (10^{-8} magnitude error or less).
Reference(s)	[RAPP2], [GN72]

7.3.3.7 Molodensky-Badekas transformation (CFR convention)

Table 7.19 — Molodensky-Badekas STT (CFR convention)

Element	Definition
STT label	CF_7_PLUS_3_PARAMETER
STT code	7
Name(s)	Molodensky-Badekas 7+3-parameter transformation (CFR convention)

Element	Definition
Description	A transformation between a local ORM and a global ORM with an origin displacement, and with both a small-scale adjustment and small angle approximation of the rotation matrix, with angles in the CFR convention, and centred at the initial point (or datum origin) of the local ORM.
Dimension	3D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. ω_1: the source $\boldsymbol{x}$-axis angle of rotation in radians in the CFR convention. ω_2: the source $\boldsymbol{y}$-axis angle of rotation in radians in the CFR convention. ω_3: the source $\boldsymbol{z}$-axis angle of rotation in radians in the CFR convention. Δs: the scale difference from unity (unitless). x_0: the x-component of the initial point in metres. y_0: the y-component of the initial point in metres. z_0: the z-component of the initial point in metres.
STT constraints	1) ω_1, ω_2 and, ω_3 are small rotations (magnitude less than $2\text{x}10^{-4}$ radians) in the CFR convention, 2) Δs is a small adjustment of scale ($\lvert\Delta s\rvert < 10^{-5}$).
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} + \begin{bmatrix} \Delta s & \omega_3 & -\omega_2 \\ -\omega_3 & \Delta s & \omega_1 \\ \omega_2 & -\omega_1 & \Delta s \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = (1 - \Delta s) \begin{bmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} \right) + (1 - \Delta s) \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}_{\mathrm{S}}$
Note(s)	1) If $(x_0, y_0, z_0) = (0,0,0)$, this STT is equivalent to the CF_7_PARAMETER STT. 2) The parameters are fit to empirical data so as to minimize residual error. The intent of using this formulation with the initial point of the local ORM is to produce a smaller residual error. In some treatments, the initial point is termed the pivot point. 3) The factor $(1 - \Delta s)$ approximates $1/(1 + \Delta s)$ with absolute error <10^{-10} if $\lvert\Delta s\rvert < 10^{-5}$. The matrix in the formulation is the Bursa-Wolf small angle approximation of a rotation matrix. When multiplied with its transverse, the matrix product approximates the identity matrix with a matrix element-wise absolute error of 10^{-8} or less. The approximation uses the results that $\lvert\omega - \sin(\omega)\rvert < \omega^3/6$ and $\lvert 1 - \cos(\omega)\rvert < \omega^2/2$ when ω is expressed in radians. When $\omega = 1''$, the error multiplied by the radius of the Earth is less than 75 µm. 4) The inverse formulation is approximate as it drops terms of magnitude 10^{-8} or less. (See notes for CF_7_PARAMETER STT.)
Reference(s)	[GN72]

7.3.3.8 General rotate-scale-translate transformation 3D

Table 7.20 — General rotate-scale-translate 3D STT

Element	Definition
STT label	ROTATE_SCALE_TRANSLATE

Element	Definition
STT code	8
Name(s)	General rotate-scale-translate transformation 3D
Description	A scaled rotation followed by a translation.
Dimension	3D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. $\left.\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right\}$: matrix $\boldsymbol{M}$ coefficients (unitless). $\boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}}$: the scale factor (unitless).
STT constraints	1) $\boldsymbol{M}$ is a rotation matrix: $\boldsymbol{M}^{-1} = \boldsymbol{M}^{T}$ and $\det(\boldsymbol{M}) = 1$, 2) $\boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}} > 0$
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} + \boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}} \boldsymbol{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$ where: $\boldsymbol{M} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \frac{1}{\boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}} s} \boldsymbol{M}^{-1} \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} \right)$
Note(s)	1) The most general form of a 3D similarity transformation. 2) Used in computer graphics applications.
Reference(s)	None

7.3.3.9 General rotate-scale-translate transformation 2D

Table 7.21 — General rotate-scale-translate 2D STT

Element	Definition
STT label	ROTATE_SCALE_TRANSLATE_2D
STT code	9
Name(s)	General rotate-scale-translate transformation 2D
Description	A scaled rotation followed by a translation.
Dimension	2D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. $\left.\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}\right\}$: matrix $\boldsymbol{M}$ coefficients (unitless). $\boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}}$: the scale factor (unitless).

Element	Definition
STT constraints	1) $\boldsymbol{M}$ is a rotation matrix: $\boldsymbol{M}^{-1} = \boldsymbol{M}^{T}$ and $\det(\boldsymbol{M}) = 1$, 2) $\boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}} > 0$
STT formulation	$\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} + \boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}} \boldsymbol{M} \begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{S}}$ where: $\boldsymbol{M} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$
STT inverse formulation	$\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{S}} = \frac{1}{\boldsymbol{\sigma}_{\mathrm{T}\leftarrow\mathrm{S}}} \boldsymbol{M}^{-1} \left(\begin{bmatrix} x \\ y \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} \right)$
Note(s)	1) The most general form of a 2D similarity transformation. 2) Used in computer graphics applications.
Reference(s)	None

7.3.3.10 Homogeneous matrix 4x4 transformation 3D

Table 7.22 — Homogeneous matrix 4x4 3D STT

Element	Definition
STT label	HOMOGENEOUS_MATRIX_4X4
STT code	10
Name(s)	Homogeneous matrix 4x4 transformation 3D homogeneous transformation matrix
Description	A scaled rotation and translation in a 4x4 matrix form.
Dimension	3D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. $\left.\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right\}$: sub-matrix $\boldsymbol{M}$ coefficients (unitless).
STT constraints	The sub-matrix $\boldsymbol{M} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a scaled rotation matrix: $\det(\boldsymbol{M}) > 0$.
STT formulation(s)	$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \Delta x \\ a_{21} & a_{22} & a_{23} & \Delta y \\ a_{31} & a_{32} & a_{33} & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\mathrm{S}}$

Element	Definition
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\mathrm{S}} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & -u \\ b_{21} & b_{22} & b_{23} & -v \\ b_{31} & b_{32} & b_{33} & -w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\mathrm{T}}$ where: $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \boldsymbol{M}^{-1}$, and $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \boldsymbol{M}^{-1} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$, and $\boldsymbol{M}^{-1} = \det(\boldsymbol{M})^{-2/3}\, \boldsymbol{M}^{\mathrm{T}}$.
Note(s)	Used in robotics and computer graphics rendering applications.
Reference(s)	[OPGL]

7.3.3.11 Homogeneous matrix 3x3 transformation 2D

Table 7.23 — Homogeneous matrix 3x3 2D STT

Element	Definition
STT label	HOMOGENEOUS_MATRIX_3X3_2D
STT code	11
Name(s)	Homogeneous matrix 3x3 transformation 2D
Description	A scaled rotation and translation in a 3x3 matrix form.
Dimension	2D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. $\left.\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}\right\}$: sub-matrix $\boldsymbol{M}$ coefficients.
STT constraints	The sub-matrix $\boldsymbol{M} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a scaled rotation matrix: $\det(\boldsymbol{M}) > 0$.
STT formulation	$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} a_{11} & a_{12} & \Delta x \\ a_{21} & a_{22} & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{\mathrm{S}} = \begin{bmatrix} b_{11} & b_{12} & -u \\ b_{21} & b_{22} & -v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{\mathrm{T}}$ where: $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \boldsymbol{M}^{-1}$, $\begin{bmatrix} u \\ v \end{bmatrix} = \boldsymbol{M}^{-1} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$, and $\boldsymbol{M}^{-1} = \det(\boldsymbol{M})^{-1}\, \boldsymbol{M}^{\mathrm{T}}$.
Note(s)	Used in computer graphics rendering applications.
Reference(s)	None

7.3.3.12 Generalized Helmert transformation (PVR convention)

Table 7.24 — Generalized Helmert STT (PVR convention)

Element	Definition
STT label	PV_XYZ_ROTATE_SCALE_TRANSLATE
STT code	12
Name(s)	Generalized Helmert 7-parameter transformation (PVR convention)
Description	General 3D similarity transformation with principal axis rotations in ***x-y-z*** body-fixed equivalent order, using the PVR convention, including normal principal rotation matrices and rotation angle values measured using right-hand-rule
Dimension	3D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. ω_1: the source $\boldsymbol{x}$-axis angle of rotation in radians in the PVR convention. ω_2: the source $\boldsymbol{y}$-axis angle of rotation in radians in the PVR convention. ω_3: the source $\boldsymbol{z}$-axis angle of rotation in radians in the PVR convention. Δs: the scale difference from unity (unitless).
STT constraints	$\Delta s > -1$
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} + (1+\Delta s)\boldsymbol{R}_X\langle\omega_1\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \circ \boldsymbol{R}_Z\langle\omega_3\rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$ where: $\boldsymbol{R}_X\langle\omega_1\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \circ \boldsymbol{R}_Z\langle\omega_3\rangle =$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & -\sin(\omega_1) \\ 0 & \sin(\omega_1) & \cos(\omega_1) \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & -\cos(\omega_2)\sin(\omega_3) & \sin(\omega_2) \\ \cos(\omega_1)\sin(\omega_3) + \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) & \cos(\omega_1)\cos(\omega_3) - \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) & -\sin(\omega_1)\cos(\omega_2) \\ \sin(\omega_1)\sin(\omega_3) - \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) & \sin(\omega_1)\cos(\omega_3) + \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \left(\frac{1}{(1+\Delta s)}\right)\boldsymbol{R}_Z^{-1}\langle\omega_3\rangle \circ \boldsymbol{R}_Y^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_X^{-1}\langle\omega_1\rangle \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}}\right)$ where: $\boldsymbol{R}_Z^{-1}\langle\omega_3\rangle \circ \boldsymbol{R}_Y^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_X^{-1}\langle\omega_1\rangle =$ $\begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & -\sin(\omega_2) \\ 0 & 1 & 0 \\ \sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & \sin(\omega_1) \\ 0 & -\sin(\omega_1) & \cos(\omega_1) \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & \cos(\omega_1)\sin(\omega_3) + \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) & \sin(\omega_1)\sin(\omega_3) - \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) \\ -\cos(\omega_2)\sin(\omega_3) & \cos(\omega_1)\cos(\omega_3) - \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) & \sin(\omega_1)\cos(\omega_3) + \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) \\ \sin(\omega_2) & -\sin(\omega_1)\cos(\omega_2) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$
Note(s)	1) A generalization of the Helmert transformation allowing large rotations. 2) The matrices in the forward and inverse formulations are the transposes of each other.
Reference(s)	[GN72]

7.3.3.13 Generalized Helmert transformation (CFR convention)

Table 7.25 — Generalized Helmert STT (CFR convention)

Element	Definition
STT label	CF_ZYX_ROTATE_SCALE_TRANSLATE
STT code	13
Name(s)	Generalized Helmert 7-parameter transformation (CFR convention)
Description	General 3D similarity transformation with principal axis rotations in ***z-y-x*** body-fixed equivalent order, using the CFR convention, including both inverted principal rotation matrices and negated rotation angle values measured using right-hand-rule
Dimension	3D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. ω_1: the source $\boldsymbol{x}$-axis angle of rotation in radians in the CFR convention. ω_2: the source $\boldsymbol{y}$-axis angle of rotation in radians in the CFR convention. ω_3: the source $\boldsymbol{z}$-axis angle of rotation in radians in the CFR convention. Δs: the scale difference from unity (unitless).
STT constraints	$\Delta s > -1$
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T\leftarrow S}} + (1+\Delta s)\boldsymbol{R}_{\boldsymbol{Z}}^{-1}\langle\omega_3\rangle \circ \boldsymbol{R}_{\boldsymbol{Y}}^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_{\boldsymbol{X}}^{-1}\langle\omega_1\rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$ where: $\boldsymbol{R}_{\boldsymbol{Z}}^{-1}\langle\omega_3\rangle \circ \boldsymbol{R}_{\boldsymbol{Y}}^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_{\boldsymbol{X}}^{-1}\langle\omega_1\rangle =$ $\begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & -\sin(\omega_2) \\ 0 & 1 & 0 \\ \sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & \sin(\omega_1) \\ 0 & -\sin(\omega_1) & \cos(\omega_1) \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & \cos(\omega_1)\sin(\omega_3) + \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) & \sin(\omega_1)\sin(\omega_3) - \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) \\ -\cos(\omega_2)\sin(\omega_3) & \cos(\omega_1)\cos(\omega_3) - \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) & \sin(\omega_1)\cos(\omega_3) + \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) \\ \sin(\omega_2) & -\sin(\omega_1)\cos(\omega_2) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \left(\frac{1}{(1+\Delta s)}\right)\boldsymbol{R}_X\langle\omega_1\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \circ \boldsymbol{R}_Z\langle\omega_3\rangle \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T\leftarrow S}}\right)$ where: $\boldsymbol{R}_X\langle\omega_1\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \circ \boldsymbol{R}_Z\langle\omega_3\rangle =$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & -\sin(\omega_1) \\ 0 & \sin(\omega_1) & \cos(\omega_1) \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & -\cos(\omega_2)\sin(\omega_3) & \sin(\omega_2) \\ \cos(\omega_1)\sin(\omega_3) + \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) & \cos(\omega_1)\cos(\omega_3) - \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) & -\sin(\omega_1)\cos(\omega_2) \\ \sin(\omega_1)\sin(\omega_3) - \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) & \sin(\omega_1)\cos(\omega_3) + \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$
Note(s)	1) A generalization of the Helmert transformation allowing large rotations. 2) The matrices in the forward and inverse formulations are the transposes of each other. 3) This is the form of 3D RT specification in the first edition of the SRM.
Reference(s)	[GN72]

7.3.3.14 Tait-Bryan *z-y-x* rotate-translate transformation

Table 7.26 — Tait-Bryan *z-y-x* rotate-translate STT

Element	Definition
STT label	TAIT_BRYAN_ZYX
STT code	13
Name(s)	Tait-Bryan ***z-y-x*** rotate-translate transformation world to entity transformation
Description	A principal axis rotation in ***z-y-x*** order, including normal principal rotation matrices and rotation angle values measured using right-hand-rule (see 6.6.4.4).
Dimension	3D
STT parameters	Δx: the x-component of the displacement from target origin to source origin in metres. Δy: the y-component of the displacement from target origin to source origin in metres. Δz: the z-component of the displacement from target origin to source origin in metres. ω_1: the source ***x***-axis (roll) angle of rotation in radians. ω_2: the source ***y***-axis (pitch) angle of rotation in radians. ω_3: the source ***z***-axis (yaw) angle of rotation in radians.
STT constraints	none
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T \leftarrow S}} + \boldsymbol{R}_Z\langle\omega_3\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \circ \boldsymbol{R}_X\langle\omega_1\rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$ where: $\boldsymbol{R}_Z\langle\omega_3\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \circ \boldsymbol{R}_X\langle\omega_1\rangle =$ $\begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & -\sin(\omega_1) \\ 0 & \sin(\omega_1) & \cos(\omega_1) \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) - \cos(\omega_1)\sin(\omega_3) & \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) + \sin(\omega_1)\sin(\omega_3) \\ \cos(\omega_2)\sin(\omega_3) & \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) + \cos(\omega_1)\cos(\omega_3) & \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) - \sin(\omega_1)\cos(\omega_3) \\ -\sin(\omega_2) & \sin(\omega_1)\cos(\omega_2) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \boldsymbol{R}_X^{-1}\langle\omega_1\rangle \circ \boldsymbol{R}_Y^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_Z^{-1}\langle\omega_3\rangle \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T \leftarrow S}} \right)$ where: $\boldsymbol{R}_X^{-1}\langle\omega_1\rangle \circ \boldsymbol{R}_Y^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_Z^{-1}\langle\omega_3\rangle =$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & \sin(\omega_1) \\ 0 & -\sin(\omega_1) & \cos(\omega_1) \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & -\sin(\omega_2) \\ 0 & 1 & 0 \\ \sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & \cos(\omega_2)\sin(\omega_3) & -\sin(\omega2) \\ \sin(\omega_1)\sin(\omega_2)\cos(\omega_3) - \cos(\omega_1)\sin(\omega_3) & \sin(\omega_1)\sin(\omega_2)\sin(\omega_3) + \cos(\omega_1)\cos(\omega_3) & \sin(\omega_1)\cos(\omega_2) \\ \cos(\omega_1)\sin(\omega_2)\cos(\omega_3) + \sin(\omega_1)\sin(\omega_3) & \cos(\omega_1)\sin(\omega_2)\sin(\omega_3) - \sin(\omega_1)\cos(\omega_3) & \cos(\omega_1)\cos(\omega_2) \end{bmatrix}$
Note(s)	1) The angle parameters are known by the following names: ω_1: roll or bank or tilt ω_2: pitch or elevation angle ω_3: heading or yaw or azimuth 2) The matrices in the forward and inverse formulations are the transposes of each other. 3) This is equivalent to a world to entity transformation in the IEEE 1278.1-2012 Standard.

Element	Definition
Reference(s)	[DIS2012]

7.3.3.15 Non-Greenwich prime meridian *z* rotate-translate transformation

Table 7.27 — Non-Greenwich prime meridian *z* rotate-translate STT

Element	Definition
STT label	PV_Z_ROTATE_TRANSLATE
STT code	14
Name(s)	Non-Greenwich prime meridian *z* rotate-translate transformation longitudinal shift and origin translation
Description	*z*-axis rotation in the PVR convention with origin translation. This transformation is used for Earth oblate ellipsoid ORMs with non-Greenwich prime meridians.
Dimension	3D
STT parameters	Δx: the *x*-component of the displacement from target origin to source origin in metres. Δy: the *y*-component of the displacement from target origin to source origin in metres. Δz: the *z*-component of the displacement from target origin to source origin in metres. ω_3: the source *z*-axis angle of rotation in radians in the PVR convention.
STT constraints	none
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} + \boldsymbol{R}_Z\langle\omega_3\rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} + \begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \boldsymbol{R}_Z^{-1}\langle\omega_3\rangle \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} \right) = \begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\mathrm{T}\leftarrow\mathrm{S}} \right)$
Note(s)	The single *z*-axis rotation is useful for the specification of the RT for an Earth ORM with non-Greenwich prime meridians.
Reference(s)	None

7.3.3.16 Geomagnetic *z*-*y* rotate transformation

Table 7.28 — Geomagnetic *z*-*y* rotate STT

Element	Definition
STT label	PV_ZY_ROTATE
STT code	16
Name(s)	Geomagnetic *z*-*y* rotate transformation
Description	*z*-*y*-axis rotation in the PVR convention.
Dimension	3D
STT parameters	ω_2: the source *y*-axis angle of rotation in radians in the PVR convention. ω_3: the source *z*-axis angle of rotation in radians in the PVR convention.
STT constraints	none.

Element	Definition
STT formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}} = \boldsymbol{R}_Z\langle\omega_3\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}}$ where: $\boldsymbol{R}_Z\langle\omega_3\rangle \circ \boldsymbol{R}_Y\langle\omega_2\rangle =$ $\begin{bmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & -\sin(\omega_3) & \sin(\omega_2)\cos(\omega_3) \\ \cos(\omega_2)\sin(\omega_3) & \cos(\omega_3) & \sin(\omega_2)\sin(\omega_3) \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix}$
STT inverse formulation	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{S}} = \boldsymbol{R}_Y^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_Z^{-1}\langle\omega_3\rangle \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\mathrm{T}}$ where: $\boldsymbol{R}_Y^{-1}\langle\omega_2\rangle \circ \boldsymbol{R}_Z^{-1}\langle\omega_3\rangle =$ $\begin{bmatrix} \cos(\omega_2) & 0 & -\sin(\omega_2) \\ 0 & 1 & 0 \\ \sin(\omega_2) & 0 & \cos(\omega_2) \end{bmatrix} \begin{bmatrix} \cos(\omega_3) & \sin(\omega_3) & 0 \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} \cos(\omega_2)\cos(\omega_3) & \cos(\omega_2)\sin(\omega_3) & -\sin(\omega_2) \\ -\sin(\omega_3) & \cos(\omega_3) & 0 \\ \sin(\omega_2)\cos(\omega_3) & \sin(\omega_2)\sin(\omega_3) & \cos(\omega_2) \end{bmatrix}$
Note(s)	This STT is useful in RT specifications for celestiomagnetic ORMs (See 7.5.8).
Reference(s)	None

7.4 Object reference model

7.4.1 Introduction

A set of bound RDs can be selected so as to be compatible with one and only one normal embedding. In this way, a set of bound RDs with properly constrained relationships can specify a unique normal embedding. Such a constrained set of bound RDs is termed an object reference model. Some object reference models use a set of RDs that model application-specific geometric aspects of the object-space. Of particular interest are object reference models that include an oriented surface RD that models a surface significant to the object (see 7.4.2).

A relationship between two or more bound RDs needed to ensure compatibility with a normal embedding is termed a binding constraint (see 7.4.3). Object reference models that use the same set of RD primitives and the same binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification. If the bound RDs of an object reference model are compliant with the RD set and binding constraints of a particular object reference model template, then the object reference model is said to realize that template (see 7.4.4).

A set of standardized object reference model templates are defined in this International Standard (see 7.4.5). Realizations of these templates are specified in Annex E.

7.4.2 Definition

A normal embedding and an RD binding are *compatible* if the normal embedding image of the RD primitive is coincident with the points (and direction or orientation, as applicable) of the measured or constructed entity of the RD binding.

EXAMPLE 1 The constructed point $\boldsymbol{p}$ in object-space to which RD ORIGIN_3D is bound is compatible with normal embedding $\boldsymbol{E}$ if and only if $\boldsymbol{E}(\mathbf{0}) = \boldsymbol{p}$.

EXAMPLE 2 The directed line constructed in object-space to which RD X_AXIS_3D is bound is compatible with a normal embedding $\boldsymbol{E}$ if and only if both the locus of the x-axis image under $\boldsymbol{E}$ coincides with the directed line, and increasing x-axis values of $\boldsymbol{E}$ advance in the forward direction of the line.

An *object reference model* (ORM) for a spatial object is a set of bound RDs for which there exists exactly one normal embedding that is compatible with each bound RD in the set. In the 3D case, this unique embedding shall also be right-handed.

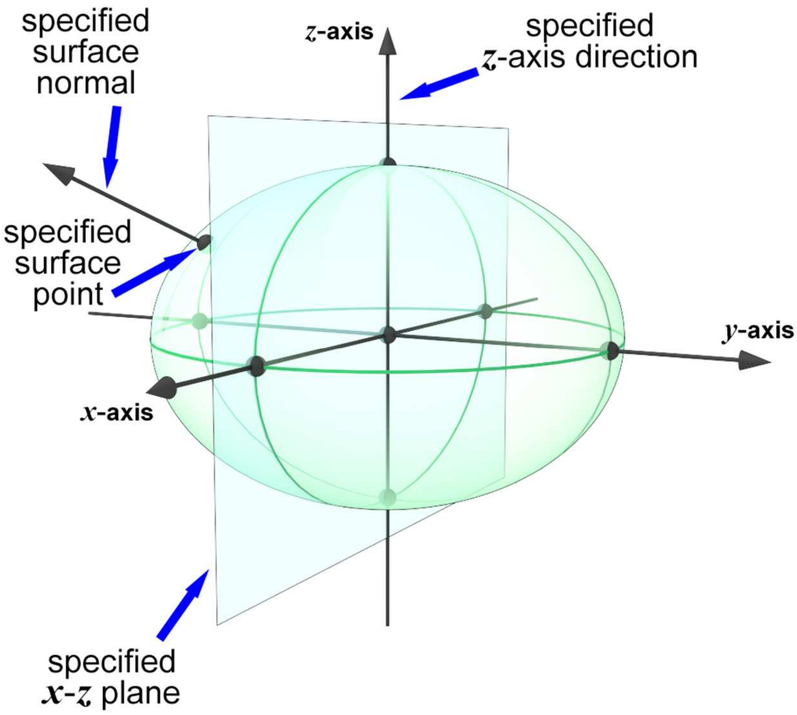


Figure 7.6 — Oblate ellipsoid ORMT binding

EXAMPLE 3 An object reference model of the Earth is created from three reference datums: an oblate ellipsoid reference datum (with specified parameter values a and b), a z-axis reference datum and an xz-plane reference datum (see Figure 7.6). These reference datums are bound as follows:

a) The oblate ellipsoid reference datum is bound to a constructed ellipsoid in object-space as in 7.2.5 Example 2 with major and minor semi-axis values a and b metres, respectively.

b) The z-axis reference datum is bound to a constructed line in object-space. This constructed line is selected to coincide with the axis of rotation of the constructed ellipsoid to ensure the existence of a compatible normal embedding. The binding of the z-axis origin is determined in c).

c) The equatorial plane of the constructed ellipsoid determines the xy-plane of any resulting spatial coordinate system. The intersection of the equatorial plane of the constructed ellipsoid with the z-axis constructed line determines the origin and the z-axis of the resulting spatial coordinate system. However, these two reference datum bindings alone do not fully determine the direction of the x-axis of the resulting spatial coordinate system.

d) The xz-plane reference datum then is bound to a constructed plane in object-space. This constructed plane is selected to contain the z-axis constructed line to ensure the existence of a compatible normal embedding.

e) The z-axis constructed line divides the xz-constructed plane into two half-planes. One half-plane is designated as the x-positive half-plane. The intersection of the equatorial plane of the constructed ellipsoid with this x-positive half-plane determines the x-axis of the spatial coordinate system and its direction. Since there is one and only one y-axis choice that is right-handed, a compatible normal embedding is uniquely determined by these three reference datum bindings.

An ORM is *object-fixed* if each of its RD bindings is object-fixed, otherwise it is termed *object-dynamic*. The object-fixed definition assumes that the object itself is not changing in time by an amount significant for the accuracy and time scale of an application. The normal embedding determined by an ORM is, correspondingly, either an *object-fixed embedding* or an *object-dynamic embedding*.

EXAMPLE 4 The Sun and the gas giants Jupiter, Saturn, Uranus, and Neptune are not rigid. The ORM specified for the Sun uses RD bindings defined in part by ephemeris and is thus an object-dynamic ORM. However, in the case of the ORMs specified for the gas giants, the ORMs are object-fixed with respect to the magnetic field of the planet.

An ORM is often selected to contain an RD of category oriented surface that corresponds to a physical or conceptual surface significant to the modelled spatial object. An RD is chosen and its position with respect to the object is bound so that the RD instance is a “best fit” to the object in some application-specific sense. If the RD surface is “fitted” to a specific part of the object surface, the ORM is termed a *local model*. If the RD is selected to best fit the entire surface, the ORM is termed a *global model*.

An ORM may also contain an RD for the purpose of providing CS parameter values (see 8.3.2.2). The radius of a sphere RD or the semi-axis values of an ~~oblate~~ ellipsoid RD may be used for this purpose.

An *Earth reference model* (ERM) is an ORM for which the spatial object is the Earth.

EXAMPLE 5 If the object is a dense planet or moon, an ORM containing an oblate ellipsoid RD or sphere RD is usually selected to model all or part of the general shape of the planet.

7.4.3 Binding constraint

In an ORM, the existence of a unique and compatible normal embedding depends on establishing certain geometric relationships, termed binding constraints, when the RDs are bound.

A *binding constraint* is a relationship in object-space between the constructed entities of two or more bound RDs, or a size relationship between a reference datum primitive and its corresponding constructed entity. Binding constraint relationships used in this International Standard include:

a) containment of a point in a curve or a surface.

b) containment of a curve in a surface.

c) coincidence of a line with an axis of symmetry of a surface.

d) in the 3D case, the right-handedness of sets of directed lines or oriented planes, or

e) distance measurement in object-space between points.

EXAMPLE The object reference model in 7.2.5 Example 1 includes the following three binding constraints:

a) The oblate ellipsoid reference datum is bound to a constructed ellipsoid that has major semi-axis length of a metres and minor semi-axis length of b metres.

b) The z-axis reference datum is bound to an object-space constructed line that coincides with the axis of rotation of the constructed ellipsoid.

c) The xz-plane reference datum is bound to an object-space constructed plane that contains the z-axis constructed line.

In this International Standard every abstract object-space has an associated length scale. The term "(scaled) metres" in a binding constraint definition shall mean "metres" in the case of physical objects and shall mean "length scaled to metres with respect to the length scale of object-space" in the case of abstract objects.

7.4.4 ORM template

ORMs that have the same set of RD primitives and that have the same binding constraints are abstracted in the notion of an ORM template. An ORM template specifies how certain sets of RD primitives may be bound to ensure that the resulting set of bound RDs forms an ORM. An ORM template can be used to conveniently specify multiple ORMs.

An *ORM template* (ORMT) is a set of RDs and binding constraints such that, for a given object-space, whenever the RDs in the set are bound in compliance with the set of binding constraints, then that bound set of RDs forms an ORM.

An ORM is a *realization* of an ORMT if

1) the RDs of the ORM match the RD set of the ORMT, and

2) the RD bindings of the ORM are compliant with the binding constraints of the ORMT.

This International Standard specifies a set of ORMTs for 2D and 3D position-space in Tables 7.31 and 7.32. The specification elements are defined in Table 7.29. Additional ORMTs may be specified by registration in accordance with 13.3.4. Table 7.30 is a directory of ORMT specification tables.

Table 7.29 — ORMT specification elements

Element		**Definition**
ORMT label		The label for the ORMT (see 13.2.2).
ORMT code		The code for the ORMT (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
ORMT specification	**Description**	A description of an ORM realization of the template.
	RD set	A list of RDs in the set.
	Binding constraints	Binding constraints.
	Notes	Optional notes.

Table 7.30 — ORMT specification directory

Position-space dimension	**Table number**
2D	Table 7.31
3D	Table 7.32

Table 7.31 — 2D ORMT specifications

ORMT label	ORMT code	ORMT specification
BI_AXIS_ORIGIN_2D	1	**Description:** x- and y-axes determined by directed perpendicular lines passing through the origin. **RD set:** RD 1) RD ORIGIN_2D RD 2) RD X_AXIS_2D RD 3) RD Y_AXIS_2D **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2 and the constructed directed line bound to RD 3. BC 2) The constructed directed lines bound to RD 2 and RD 3 shall be perpendicular. **Notes:** 1) The constructed point bound to RD 1 determines the origin of the normal embedding. 2) The perpendicular directed lines passing through the origin uniquely determine the x-axis and y-axis of the normal embedding.

Table 7.32 — 3D ORMT specifications

ORMT label	ORMT code	ORMT specification
SPHERE	2	**Description:** 3D sphere with designated directional axis and xz-plane. **RD set:** RD 1) The sphere RD with radius r. RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The constructed directed line bound to RD 2 shall contain the centre of the constructed sphere bound to RD 1. BC 2) The constructed plane bound to RD 3 shall contain the constructed directed line bound to RD 2. BC 3) The radius of the constructed sphere bound to RD 1 shall be r (scaled) metres. **Notes:** 1) The centre of the constructed sphere bound to RD 1 determines the origin of the normal embedding. 2) The constructed directed line bound to RD 2 passing through the origin of the normal embedding uniquely determines the z-axis of the normal embedding. 3) The plane through the origin of the normal embedding perpendicular to the z-axis of the normal embedding determines the xy-plane of the normal embedding. The constructed plane bound to RD 3 determines the xz-plane of the normal embedding. The intersection of the constructed xz-plane with the xy-plane is the locus of the x-axis of the normal embedding. The positive side of the xz-plane is designated in the RD binding and together with the direction of the z-axis of the normal embedding determines the direction of the x-axis of the normal embedding. 4) The line perpendicular to the xz-plane of the normal embedding through the origin of the embedding determines the locus of the y-axis of the normal embedding. Its direction is determined by the right-handedness of the normal embedding. 5) The binding constraint BC 3 is required for length compatibility with a normal embedding.

ORMT label	ORMT code	ORMT specification
OBLATE_ELLIPSOID	3	**Description:** Oblate ellipsoid with designated minor axis direction and xz-plane. **RD set:** RD 1) The oblate ellipsoid RD with major semi-axis a and minor semi-axis b RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The spatial plane RD 3 shall contain the minor axis of the spatial oblate ellipsoid RD 1. BC 2) The spatial z-axis RD 2 shall coincide with the minor axis of the spatial oblate ellipsoid RD 1. BC 3) The RD 1 length of the spatial major semi-axis shall be a (scaled) metres and the length of the spatial minor semi-axis shall be b (scaled) metres. **Notes:** 1) The centre of the constructed oblate ellipsoid bound to RD 1 determines the origin of the normal embedding. 2) The constructed directed line bound to RD 2 passing through the origin (as required by BC 1) uniquely determines the z-axis of the normal embedding. 3) The z-axis of the normal embedding determines the xy-plane as the perpendicular plane through the origin of the normal embedding. BC 2 requires the bound xz-plane to contain the z-axis. The line formed by the intersection of the bound xz-plane with the xy-plane is the x-axis line. The positive side of the constructed plane bound to RD 3 determines the xz-plane and together with the direction of the z-axis determines the direction of the x-axis. 4) The y-axis is determined by the required right-handedness of the normal embedding. 5) The binding constraint BC 3 is required for length compatibility with a normal embedding.

ORMT label	ORMT code	ORMT specification
PROLATE_ELLIPSOID	4	**Description:** 3D prolate ellipsoid with designated major axis direction and *xz*-plane. **RD set:** RD 1) The prolate ellipsoid RD with minor semi-axis *a* and major semi-axis *b.* RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The spatial plane shall contain the major axis of the spatial prolate ellipsoid. BC 2) The spatial *z*-axis shall coincide with the major axis of the spatial prolate ellipsoid. BC 3) The length of the spatial major semi-axis shall be *b* (scaled) metres and the length of the spatial minor semi-axis shall be *a* (scaled) metres.
TRI_AXIAL_ELLIPSOID	5	**Description:** 3D tri-axial ellipsoid with designated *z*-axis direction and *xz*-plane. **RD set:** RD 1) The tri-axial ellipsoid RD with *x*-semi-axis *a, y*-semi-axis *b,* and *z*-semi-axis *c.* RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The spatial plane shall contain the *z*-axis of the spatial tri-axial ellipsoid. BC 2) The spatial *z*-axis shall coincide with the *z*-axis of the spatial tri-axial ellipsoid. BC 3) The length of the spatial *x*-semi-axis shall be *a* (scaled) metres, spatial *y*-semi-axis shall be *b* (scaled) metres, and the length of the spatial *z*-semi-axis shall be *c* (scaled) metres.

ORMT label	ORMT code	ORMT specification
BI_AXIS_ORIGIN_3D	6	**Description:** x- and z-axes determined by directed perpendicular lines passing through the origin. **RD set:** RD 1) RD ORIGIN_3D RD 2) RD Z_AXIS_3D RD 3) RD X_AXIS_3D **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2 and the constructed directed line bound to RD 3. BC 2) The constructed directed lines bound to RD 2 and RD 3 shall be perpendicular. **Notes:** 1) The constructed point bound to RD 1 determines the origin of the normal embedding. 2) The perpendicular directed lines passing through the origin uniquely determine the z-axis and x-axis of the normal embedding. 3) The y-axis is determined by the required right-handedness of the normal embedding.

<table>
<tr><th>ORMT label</th><th>ORMT code</th><th>ORMT specification</th></tr>
<tr><td>SPHERE_ORIGIN</td><td>7</td><td>Description: Sphere with two directed perpendicular lines passing through the centre of the sphere.
RD set:
RD 1) RD ORIGIN_3D
RD 2) RD Z_AXIS_3D
RD 3) RD X_AXIS_3D
RD 4) The sphere RD with radius r.
Binding constraints:
BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2 and the constructed directed line bound to RD 3.
BC 2) The constructed directed lines bound to RD 2 and RD 3 shall be perpendicular.
BC 3) The centre of the constructed sphere bound to RD 4 shall be coincident with the point bound to RD 1.
BC 4) The radius of the constructed sphere bound to RD 4 shall be r (scaled) metres.
Notes:
1) The point bound to RD 1 determines the origin of the normal embedding.
2) The perpendicular directed lines passing through the sphere centre uniquely determine the z-axis and x-axis of the normal embedding.
3) The y-axis is determined by the required right-handedness of the normal embedding.
4) The binding constraint BC 4 is required for length compatibility with a normal embedding.
5) The sphere RD is included to provide a CS parameter value (radius). See 7.4.2.</td></tr>
</table>

ORMT label	ORMT code	ORMT specification
OBLATE_ELLIPSOID-_ORIGIN	8	**Description:** Oblate ellipsoid with designated centre, minor axis direction and xz-plane. **RD set:** RD 1) RD ORIGIN_3D RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D RD 4) The oblate ellipsoid RD with major semi-axis a and minor semi-axis b. **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2. BC 2) The spatial plane RD 3 shall contain the constructed directed line bound to RD 2. BC 3) The minor axis of the spatial oblate ellipsoid RD 4 shall be coincident with the directed line bound to RD 2 and the ellipsoid centre shall coincide with the point bound to RD 1. BC 4) The RD 4 length of the spatial major semi-axis shall be a (scaled) metres and the length of the spatial minor semi-axis shall be b (scaled) metres. **Notes:** 1) The centre of the constructed sphere bound to RD 1 determines the origin of the normal embedding. 2) The constructed directed line bound to RD 2 passing through the origin uniquely determines the z-axis of the normal embedding. 3) The z-axis of the normal embedding determines the xy-plane as the perpendicular plane through the origin of the normal embedding. BC 2 requires the bound xz-plane to contain the z-axis. The line formed by the intersection of the bound xz-plane with the xy-plane is the x-axis line. The positive side of the constructed plane bound to RD 3 determines the xz-plane and together with the direction of the z-axis determines the direction of the x-axis. 4) The y-axis is determined by the required right-handedness of the normal embedding. 5) The binding constraint BC 4 is required for length compatibility with a normal embedding. 6) The oblate ellipsoid RD is included to provide CS parameter values (major and minor semi-axes). See 7.4.2.

ORMT label	ORMT code	ORMT specification
TRI_PLANE	9	**Description:** Origin determined by the intersection of three planes. **RD set:** RD 1) RD XZ_PLANE_3D RD 2) RD XY_PLANE_3D RD 3) RD YZ_PLANE_3D **Binding constraints:** BC 1) The spatial planes shall be pair-wise perpendicular. BC 2) The collective formation of the three planes shall be right-handed. **Notes:** 1) The intersection of all three planes RD 1, RD 2, and RD 3 determine the origin of the normal embedding. 2) The intersection of the xz-plane RD 1 and the xy-plane RD 2 determine the line of the x-axis. 3) The positive side designations of the two planes RD 1 and RD 2 together with the right-handedness requirement determines the positive x-axis direction. The directed line and the origin point determine the x-axis of the normal embedding. 4) The z- and y-axes of the normal embedding are similarly determined. 5) BC 2 is required for compatibility with the right-handedness requirement.

The specification of an ORMT does not determine a normal embedding. A normal embedding is determined when an ORMT is realized as an ORM.

The methods and techniques of binding the RDs of an ORMT realization draw from disciplines ranging from geometry to astronomy, surveying, geophysics, and satellite geodesy. In general, there are many ways to realize an ORMT for a spatial object. Techniques and methodologies for binding RD components are outside of the scope of this International Standard. The ORM concept is designed to be general enough to encompass these many application domains. As an illustration of the generality of this concept, the following Example outlines a method used in geodesy to define the RD bindings of an ORMT OBLATE_ELLIPSOID realization.

EXAMPLE The North American Datum 1927 may be specified as a realization of the ORMT OBLATE_ELLIPSOID (see Table 7.32). The oblate ellipsoid RD component is RD CLARKE 1866 in Table D.2. The binding of this RD and the other two RD components in the Earth object-space may be defined as follows:

a) the direction of the RD Z_AXIS_3D is identified as the north direction of the Earth's rotational axis,

b) a position (latitude 39°13'26,686"N) on the oblate ellipsoid RD is identified to a location in the Earth object-space (Meades Ranch, Kansas, US),

c) the direction of the surface normal to the ellipsoid at the identified point is specified (ξ = -1,32" η = 1,93"), and

d) the direction of the positive xz-plane normal is indirectly determined by specifying the longitude of Meades Ranch (98°32'30,506"W).

Items a) through c) determine a unique oblate ellipsoid surface in the object-space of the Earth. The equatorial plane of the oblate ellipsoid is (by compatibility with the oblate ellipsoid surface generating function) the xy-plane, and its intersection with the oblate ellipsoid axis of rotation determines the origin point and the z-axis. Specification (d) together with the origin and the xy-plane determine the x-axis. Since there is one, and only one, oriented yz-plane that is both perpendicular to the xz-plane and right-handed compatible, the right-handed normal embedding is uniquely determined (see Figure 7.6).

This Example is based on a published specification of the North American Datum 1927. However, the methodology used to select the point in b) and determine the surface normal direction c) at that point was a complex process involving a mathematical best fit of the Clarke 1866 ellipsoid to a network of geodetic survey control points spanning the continental United States.

7.4.5 Standardized ORMs

The ORMs specified in this International Standard, are ORMT realizations. Standardized ORMs shall include:

a) a specification of the spatial object and optionally a region in object-space,

b) a specification of the ORM template,

c) a specification of the ellipsoid RD (if any), and

d) the binding year if the spatial object is a physical object.

A standardized ORM does not include a specification of the binding of its RD components. The binding specification is not directly needed for the scope of this International Standard. An ORM indirectly designates a unique normal embedding. A specification of an ORM normal embedding is important when two or more ORMs have been specified for the same spatial object. To inter-convert between spatial relationships with respect to two different normal embeddings, one normal embedding shall be expressed in terms of the other normal embedding by means of a similarity transformation, or both shall be expressed in terms of a third (reference) normal embedding by means of a similarity transformation for each ORM with respect to a third (reference) ORM.

7.4.6 Reference ORMs and reference transformations

If two or more object-fixed ORMs for the same object are specified (or registered), one of the ORMs shall be designated as the *reference ORM* for that object.

A *reference transformation* (RT) for an ORM is a similarity transformation from the embedded frame of the ORM to the embedded frame of the reference ORM for that object, ORM_R. The reference transformation for an ORM, ORM_S, shall be denoted by $\boldsymbol{H}_{\text{R}\leftarrow\text{S}}$ (see Table 10.1).

For 3D ORMs, a reference transformation shall be specified as an STT together with the values that correspond to the STT parameters. In the case of dynamic ORMs (see 7.5), parameter values may be functions of time in a specified temporal coordinate system. Standardized object-fixed ORMs shall specify at least one reference transformation.

Some ERMs that are specified in Table E.5 are based on local geodetic datums. Historic local geodetic datums may exhibit distortions with respect to the reference ERM. In these cases, an RT specified by the ERM is an approximation based on empirical measurements. In some of these cases, similarity transformation parameters for approximations based on different sets of measurements and/or sub-regions appear as multiple RT entries in Table E.6. In those cases, the RT labels shall share a common prefix derived from the name of the related local geodetic datum.

NOTE 1 In some cases a scale adjustment is needed to account for differing length scales in abstract object-space. The STTs PV_7_PARAMETER, CF_7_PARAMETER, CF_7_PLUS_3_PARAMETER, ROTATE_SCALE_TRANSLATE, and CF_ZYX_ROTATE_SCALE_TRANSLATE provide either a scale factor $\boldsymbol{\sigma}$ parameter or a small-scale adjustment parameter $(1 + \Delta s)$. Theoretically, $\boldsymbol{\sigma} = 1$ or $\Delta s = 0$ for normal embeddings of a physical object-space. In practice, when the embeddings are indirectly determined by the RD bindings of an ORM, the determination of the parameters is an approximation that is the result of a mathematical best fit and the scale parameter (σ or Δs) is used to provide an additional degree of freedom to achieve the desired accuracy in the region of interest. In the case of the object Earth, published values for Δs are typically one part per million in magnitude (10^{-6}) or smaller.

A directory of reference ORMs is provided in Table E.1. The reference ORM for the Earth is ORM WGS_1984. This ORM is an Earth-fixed global model. The actual binding definition of ORM WGS_1984 (see [NGA36]) is a realization of ORMT BI_AXIS_ORIGIN_3D. However, for purposes of this International Standard, the RD WGS_1984 is associated with this ORM, so that its ORM specification is a realization of ORMT OBLATE_ELLIPSOID. The equivalent binding for this ORMT is as follows: The centre of RD WGS_1984 is bound to the centre of mass for the whole Earth, including oceans and atmosphere; the RD Z_AXIS_3D is bound to the line passing through the centre of mass in the direction of the IERS reference pole (the North pole); and the RD XZ_PLANE_3D is bound to the plane that contains the IERS reference pole, centre of mass and the IERS reference meridian (the Greenwich meridian), with the x-axis extending from the centre of mass to the IERS reference meridian.

7.4.7 ORM specifications

The elements of an ORM specification are defined in Table 7.33. Standardized ORMs are specified in Annex E. Additional ORMs may be specified by registration in accordance with Clause 13.

Table 7.33 — ORM specification elements

Element	Definition
ORM label	The label for the ORM (see 13.2.2).
ORM code	The code for the ORM (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Published name	The name(s) given to the concept embodied in this ORM in the reference(s).
Reference ORM	The label of the reference ORM for this object. If this ORM is the reference ORM for this object, then this specification element shall contain the phrase “This is the reference ORM for” followed by the object name. If no object-fixed ORM specification exits, this specification element shall contain the string “none”.
Binding information	Case: Object-fixed ORM for a physical object If the spatial object is a physical object, the date that the ORM RD components were bound in object-space. If the ORM is based on ORMT OBLATE_ELLIPSOID, OBLATE_ELLIPSOID_ORIGIN, SPHERE, or SPHERE_ORIGIN, a significant location contained in the x-positive xz-half-plane of the normal embedding shall be specified. In cases where the spatial object is the Earth, this location shall be understood to be Greenwich, UK, unless otherwise specified. Case: Dynamic ORM for a physical object If the ORM is based on ORMT BI_AXIS_ORIGIN_3D and if the ORM binding complies with a standardized object binding rule set, the label of that object binding rule set (see 7.5). If the ORM is a time-fixed (object-fixed) instance of a dynamic ORM, the date that the ORM RD components were bound in object-space. Case: ORM for an abstract object The string “none”. All cases: Optional binding notes.

Element	Definition
Region	The approximate subset of object-space to which the model applies, expressed as either a spatial extent or the description as specified in the reference.
ORMT label	The label of the ORM template for this ORM.
RD parameterization	The label of the ellipsoidal RD, if any; otherwise “n/a”.
References	The references (see 13.2.5), or “none” if defined in this International Standard.

For each object-fixed ORM there shall be specified one or more RTs that transform the ORM to the reference ORM of the ORM spatial object. The elements of an RT specification are defined in Table 7.34. Standardized RTs are specified in Annex E. Additional RTs may be specified by registration in accordance with Clause 13. The standardized ORM for an abstract space of a given dimension specifies only an identity RT. Graphical applications use *ad hoc* transformations to scale, rotate, and translate one abstract space with respect to another as needed in an application. The API (Clause 11) provides support for non-standardized RTs.

Table 7.34 — Reference transformation specification elements

Element	Definition
ORM label	The label of the standardized ORM that this RT transforms.
RT label	The label for the RT (see 13.2.2).
RT code	The code for the RT (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
RT region	A non-normative description of the extent and/or the spatial boundaries of the region for which this reference transformation is applicable. Angles may be expressed in arc degrees (°) in order to avoid a loss of precision.

Element	Definition
STT label and parameter values	The label of the STT that is used to specify the $\boldsymbol{H}_{\mathrm{R \leftarrow S}}$ transformation. The values of the STT parameters shall be specified by value or by reference (see 13.2.5) using the STT parameter symbols. If by value, the values of the STT parameters specifying the reference transformation $\boldsymbol{H}_{\mathrm{R \leftarrow S}}$ (see Table 10.1) shall be specified. These values may be followed by a measurement/modelling error estimate expressed in one of the following forms: a) : assumed precise b) : σ = <standard error> c) : ±<tolerance> d) no error information following a parameter value indicates that the error estimate is unknown or unattainable. EXAMPLE $\Delta x = 12{:}\, \sigma x = 5,$ $\Delta y = -133{:}\, \pm 25,$ $\Delta z = 0$:assumed precise. If by reference, this specification element shall contain a citation(s) for the values of the STT parameters and error estimates. Terms appearing in the references that are cited for a value shall be enclosed in brackets ({ }). Any parameter value that is not specified in the citation(s) shall be specified as in the “by value” case. A dynamic $\boldsymbol{H}_{\mathrm{R \leftarrow S}}(t)$ transformation may specify parameter values as functions of time. Optional notes concerning parameter values may be provided. To avoid loss of precision, axis rotation angles (if applicable to the STT) may be expressed in arc seconds (“) and, in cases of a large rotation, in arc degrees (°). NOTE The axis rotation angles are converted to radians when used in appropriate STT formulations.
Date published	The date that the RT was published.
References	The references (see 13.2.5), or “none” if defined in this International Standard.

RTs for standardized ORMs are specified in Annex E. In the Annex E specification tables, the specification elements STT label and STT parameters are combined for legibility.

If a 3D ORM is the reference ORM of a spatial object, then it shall have an RT with the RT label containing the string “IDENTITY”, and the RT shall be specified using the IDENTITY STT. These reference ORM identity RTs are labelled and coded to provide uniform treatment of all object-fixed ORMs in the API (see Clause 11).

If a 3D ORM is not the reference ORM of a spatial object, and an RT is the identity transformation by intent or design of the ORM, then the RT label shall contain the string “IDENTITY_BY_DEFAULT”, and the RT shall be specified using the IDENTITY STT.

If a 3D ORM is not the reference ORM of a spatial object, and an RT of the ORM is empirically equivalent to the identity transformation, then the RT label shall contain the string “IDENTITY_BY_MEASUREMENT”, and the RT shall be specified using the TRANSLATE STT with zero for the parameter values and appropriate measurement error estimates.

NOTE 2 In the case of Earth-fixed ERM bindings axis rotations are either zero, or are very small with the following exceptions:

a) Cases for which the xz-plane does not contain Greenwich, UK have relatively large ω_3 rotations.

b) The Earth-fixed approximations of celestiomagnetic ERMs have large ω_2 and ω_3 rotations (see "GEOMAGNETIC" entries in Table 7.50).

NOTE 3 A geodetic datum that is global or geocentric is used as a basis for establishing a geocentric SRF (see 8.5.2). If geocentric coordinates are associated to a 3D linear embedding, then such a datum is conceptually equivalent to an ERM. The WGS 84 global datum (see NGA36]) is conceptually equivalent to the ORM WGS_1984 in Table E.5.

7.5 Object binding rules for ORMT BI_AXIS_ORIGIN_3D realizations

7.5.1 Object binding rule set

ORMs for planets, satellites, and other celestial bodies include object-dynamic ORMs based on ORMT BI_AXIS_ORIGIN_3D. Many of these object-dynamic ORMs share common rules for the binding of ORMT BI_AXIS_ORIGIN_3D components based on physical characteristics and spatial arrangements of the applicable celestial bodies. To facilitate uniform specification of these ORMs, the concept of an object binding rule set for an ORMT BI_AXIS_ORIGIN realization is defined.

An *object binding rule set* (OBRS) for ORMT BI_AXIS_ORIGIN_3D shall be comprised of:

a) an object binding rule set name,

b) a label and code,

c) object restrictions that delineate the object-spaces for which the object binding rules apply, and

d) a set of object binding rules for the RD components of ORMT BI_AXIS_ORIGIN_3D,

where an *object binding rule* for an ORMT is an object-space specific restriction for the binding of a single RD in the RD set of the ORMT. The set of object binding rules (d) shall comply with the binding constraints of the ORMT.

The specification elements for an OBRS for ORMT BI_AXIS_ORIGIN_3D are defined in Table 7.35.

Table 7.35 — OBRS for ORMT BI_AXIS_ORIGIN_3D specification elements

Element	Definition
OBRS label	The label for the OBRS (see 13.2.2).
OBRS code	The code for the OBRS (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Short name	A descriptive name.
Applicable ORMTs	A list of the ORMTs to which the OBRS applies.
Object restrictions	A specification of the set of objects to which this object binding rule set applies.
Object binding rules	A specification of the binding rules.
Figures	Zero or more figures that explain and illustrate the OBRS.
References	Zero or more references (see 13.2.5).

This International Standard provides a collection of OBRS specifications as identified in Table 7.36. Additional OBRSs may be specified by registration in accordance with Clause 13.

Table 7.36 — OBRS for ORMT BI_AXIS_ORIGIN_3D specification directory

OBRS name	Table number
equatorial inertial	Table 7.37
solar ecliptic	Table 7.39
solar equatorial	Table 7.41
heliocentric Aries ecliptic	Table 7.43
heliocentric planet ecliptic	Table 7.45
heliocentric planet equatorial	Table 7.47
celestiomagnetic	Table 7.49
solar magnetic ecliptic	Table 7.51
solar magnetic dipole	Table 7.53

An OBRS name may be used to describe an ORM that is compliant with the named OBRS. Thus a "celestiomagnetic ORM" denotes an ORM realization of ORMT BI_AXIS_ORIGIN_3D for an object-space that satisfies the celestiomagnetic OBRS object restrictions and whose RD bindings comply with the celestiomagnetic OBRS object binding rule set.

Several OBRS specifications in Table 7.36 are described using one or more of the following terms: rotational northwards, vernal equinox, inertial direction, quasi-inertial direction, first point of Aries, and Aries true of date. These terms are defined as follows:

The term *rotational northwards*, as used with respect to a rotating object, shall mean in a direction making an acute angle with respect to the direction from the centre of the object to its rotational north pole.

The ecliptic plane and the plane containing the Sun and parallel to the equatorial plane of a planet intersect in a line. An *equinox* is one of the two points of intersection between this line and the orbit of the planet. The *vernal equinox* is the equinox at which the direction from the planet to the Sun begins to ascend to the northern side of the equatorial plane.

An *inertial direction* is a direction with respect to the universe that is time invariant. A *quasi-inertial direction* is a direction that changes relatively slowly with respect to the universe.

At any given epoch, the direction of the Sun at the vernal equinox is fixed with respect to the Milky Way and other (detectible) galaxies. In the case of the Earth, the *first point of Aries* is the direction from the vernal equinox to the Sun. The current first point of Aries is termed *Aries true of date*.

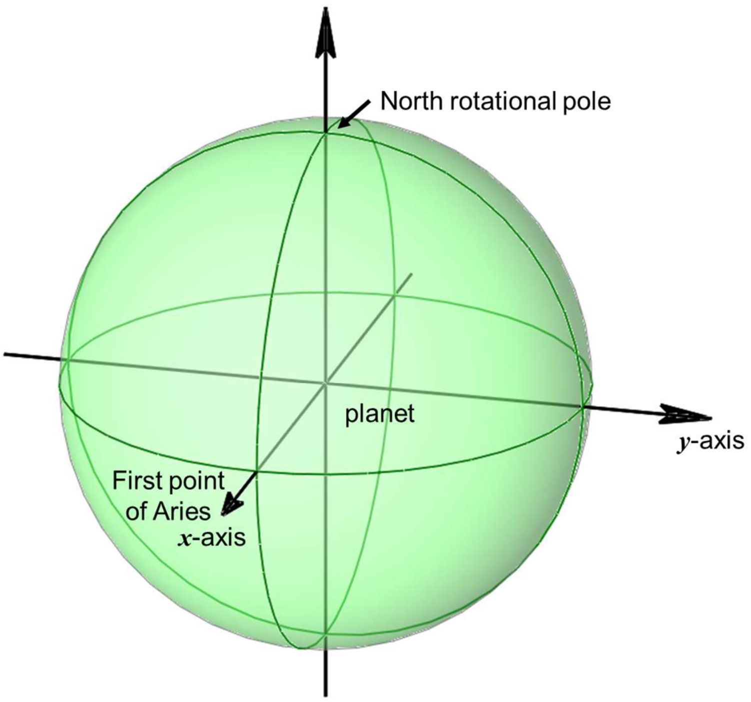
NOTE The effects of precession and nutation on the spin axis of a planet cause this direction to change over time. The change in direction of the first point of Aries is very slow (approximately one full rotation in 26 000 years).

EXAMPLE The first point of Aries at a given epoch is an inertial direction, and Aries true of date is a quasi-inertial direction.

7.5.2 Equatorial inertial

The *equatorial inertial OBRS* is specified in Table 7.37.

Table 7.37 — Equatorial inertial OBRS

Element	Value
OBRS label	EQUATORIAL_INERTIAL
OBRS code	1
Short name	equatorial inertial
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	A planet in the solar system for which the ecliptic plane is distinct from the equatorial plane.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D points in the direction of the Sun when the planet is at its vernal equinox. 3) The RD Z_AXIS_3D is parallel to the rotational axis and points in the direction of rotational northwards.
Figures	
References	[SEID]

The axis directions are quasi-inertial but vary with respect to any object-fixed ORM for the planet.

In the case of ORM EARTH_INERTIAL_J2000r0, the International Earth Rotation and Reference Systems Service (see [IERS36]) specifies a very precise transformation to the ORM WGS_1984 reference embedding with a matrix whose coefficients represent the effects of polar motion, the Earth's rotation, nutation and precession[22].

EXAMPLE The *Greenwich sidereal hour angle* $\theta_{\mathrm{GSH}}(t)$ is the angle in radians from the first point of Aries to the direction of the x-axis of ORM WGS_1984 Earth reference ORM. The Greenwich sidereal hour angle depends on the epoch of definition of the first point Aries and is a function of UTC time t elapsed from a given epoch. Approximations of this function are published in astronomical almanacs and other documents (see [SEID] or [USNOA]).

Given an ERM in the equatorial inertial OBRS, if it is assumed that the ERM z-axis and ORM WGS_1984 z-axis are coincident, then the ERM RT is specified by STT CF_ZYX_ROTATE_SCALE_TRANSLATE with dynamic parameter $\omega_3(t) = \theta_{\mathrm{GSH}}(t)$ And all other parameters set to zero.

Three equatorial inertial ERMs labelled ORM EARTH_INERTIAL_ARIES_1950, ORM EARTH_INERTIAL_J2000r0 and ORM EARTH_INERTIAL_ARIES_TRUE_OF_DATE, are specified in Table E.5. They are based on three determinations of the first point of Aries and the rotational axis. The first two have epoch-fixed inertial directions. ORM EARTH_INERTIAL_ARIES_TRUE_OF_DATE has a quasi-inertial x-axis direction that varies as a function of time.

Table 7.38 is a directory of the Earth and other celestial object equatorial inertial ORMs.

Table 7.38 — Equatorial inertial ORM directory

ORM label	Published name
EARTH_INERTIAL_ARIES_1950	Earth equatorial inertial, Aries mean of 1950
EARTH_INERTIAL_ARIES_TRUE_OF_DATE	Earth equatorial inertial, Aries true of date
EARTH_INERTIAL_J2000r0	Earth equatorial inertial, J2000.0
JUPITER_INERTIAL	Jupiter equatorial inertial
MARS_INERTIAL	Mars equatorial inertial
MERCURY_INERTIAL	Mercury equatorial inertial
NEPTUNE_INERTIAL	Neptune equatorial inertial
PLUTO_INERTIAL	Pluto equatorial inertial
SATURN_INERTIAL	Saturn equatorial inertial
URANUS_INERTIAL	Uranus equatorial inertial
VENUS_INERTIAL	Venus equatorial inertial

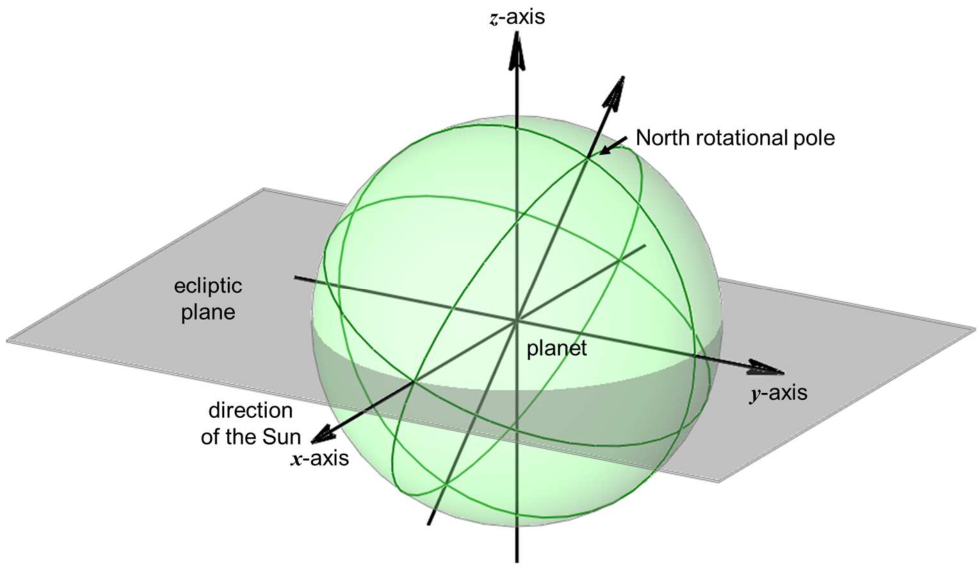
7.5.3 Solar ecliptic

The *solar ecliptic OBRS* is specified in Table 7.39. See [BHAV, 3.2.5].

Table 7.39 — Solar ecliptic OBRS

Element	Value
OBRS label	SOLAR_ECLIPTIC

[22] For near-real-time orbit determination applications, Earth orientation parameters (polar motion and Earth rotation variations) that are needed to build the matrix are predicted values. Because the driving forces that influence polar motion and Earth rotation variations are difficult to characterize, these Earth orientation predictions are performed weekly [83502T].

Element	Value
OBRS code	2
Short name	solar ecliptic
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	A planet in the solar system.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D is in the ecliptic plane and points in the direction of the Sun. 3) The RD Z_AXIS_3D is perpendicular to the ecliptic plane and points northward.
Figures	 NOTE The x-axis slowly rotates once per orbit of the planet around the Sun. The y-axis also lies in the ecliptic plane.
References	[HAPG]

EXAMPLE The *obliquity of the ecliptic* $\varepsilon(t)$ is the angle in radians from the equatorial plane to ecliptic.

The *ecliptic longitude of the Sun* $\lambda_{\odot}(t)$ is the angle in radians from the first point of Aries and the line from the centre of the Earth to the centre of the Sun. The direction to the Sun is represented by $\lambda_{\odot}(t)$.

The Greenwich sidereal hour angle $\theta_{\mathrm{GSH}}(t)$ is the angle in radians from the first point of Aries to the direction of the x-axis of ORM WGS_1984 Earth reference ORM.

$\theta_{\mathrm{GSH}}(t)$ and $\lambda_{\odot}(t)$ depend on the epoch of definition of the first point of Aries. $\theta_{\mathrm{GSH}}(t)$, $\varepsilon(t)$, and $\lambda_{\odot}(t)$ are functions of UTC time t elapsed from a given epoch. Approximations of these functions are published in astronomical almanacs and other documents (see [SEID] or [USNOA]).

Given an ERM in the solar ecliptic OBRS, the ERM RT is specified by STT CF_ZYX_ROTATE_SCALE_TRANSLATE with dynamic parameters:

$\omega_1(t) = -\varepsilon(t)$
$\omega_3(t) = \theta_{\mathrm{GSH}}(t) - \lambda_{\odot}(t)$
and all other parameters set to zero.

Table 7.40 is a directory of the Earth and other planet object solar ecliptic ORMs.

Table 7.40 — Solar ecliptic ORM directory

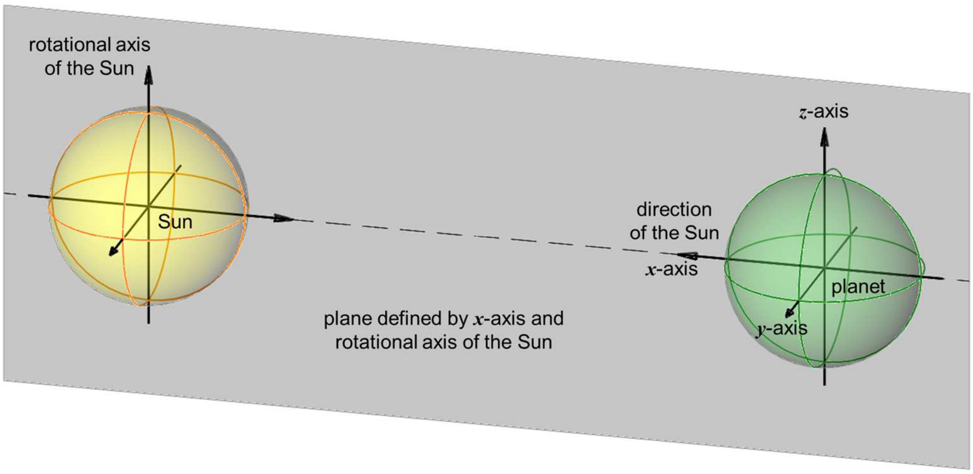
ORM label	Published name
EARTH_SOLAR_ECLIPTIC	Solar ecliptic
JUPITER_SOLAR_ECLIPTIC	Jupiter solar ecliptic

7.5.4 Solar equatorial

A *solar equatorial OBRS* is specified in Table 7.41.

Table 7.41 — Solar equatorial OBRS

Element	Value
OBRS label	SOLAR_EQUATORIAL
OBRS code	3
Short name	solar equatorial
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	A planet in the solar system for which the ecliptic plane is distinct from the equatorial plane.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D is in the ecliptic plane and points in the direction of the Sun. 3) The RD Z_AXIS_3D is perpendicular to the RD X_AXIS_3D in the plane determined by the RD X_AXIS_3D and the rotational axis of the Sun and points northward.

<table>
<tr><th>Element</th><th>Value</th></tr>
<tr><td>Figures</td><td>


NOTE The xz-plane of a solar equatorial ORM contains the rotational axis of the Sun. The x-axis slowly rotates once per orbit of the object around the Sun. The y-axis is parallel to the solar equatorial plane and points towards dusk [BHAV, 3.2.6].
</td></tr>
<tr><td>References</td><td>[CRUS]</td></tr>
</table>

Table 7.42 is a directory of the Earth and other planet solar equatorial ORMs.

Table 7.42 — Solar equatorial ORM directory

ORM label	Published name
EARTH_SOLAR_EQUATORIAL	Solar equatorial
JUPITER_SOLAR_EQUATORIAL	Jupiter solar equatorial

7.5.5 Heliocentric Aries ecliptic

The *heliocentric Aries ecliptic OBRS* is specified for a planet in Table 7.43. See [HAPG].

Table 7.43 — Heliocentric Aries ecliptic OBRS

Element	Value
OBRS label	HELIOCENTRIC_ARIES_ECLIPTIC
OBRS code	4
Short name	heliocentric Aries ecliptic
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	The Sun.

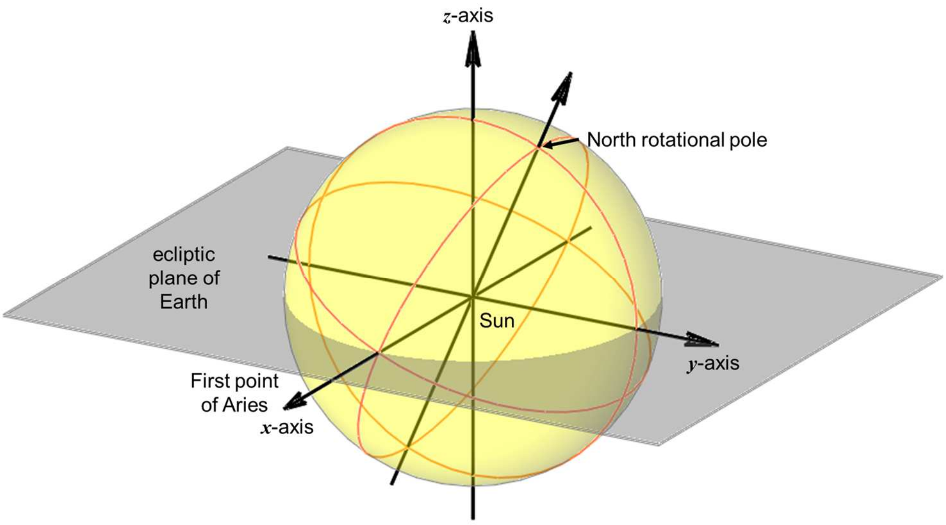
Element	Value
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the Sun. 2) The RD Z_AXIS_3D is perpendicular to the ecliptic plane of the Earth and points in the direction of rotational northwards. 3) The RD X_AXIS_3D is in the ecliptic plane of the Earth and points towards the first point of Aries.
Figures	
References	[HAPG]

Table 7.44 is a directory of heliocentric Aries ecliptic ORMs. The heliocentric Aries ecliptic axis directions are inertial for ORM HELIO_ARIES_ECLIPTIC_J2000r0 and quasi-inertial ORM HELIO_ARIES_ECLIPTIC_TRUE_OF_DATE.

Table 7.44 — Heliocentric Aries ecliptic ORM directory

ORM label	Published name
HELIO_ARIES_ECLIPTIC_J2000r0	Heliocentric Aries ecliptic, J2000.0
HELIO_ARIES_ECLIPTIC_TRUE_OF_DATE	Heliocentric Aries ecliptic, true of date

7.5.6 Heliocentric planet ecliptic

The *heliocentric planet ecliptic OBRS* is specified for a planet in Table 7.45. See [HAPG].

Table 7.45 — Heliocentric planet ecliptic OBRS

Element	Value
OBRS label	HELIOCENTRIC_PLANET_ECLIPTIC
OBRS code	5
Short name	heliocentric planet ecliptic

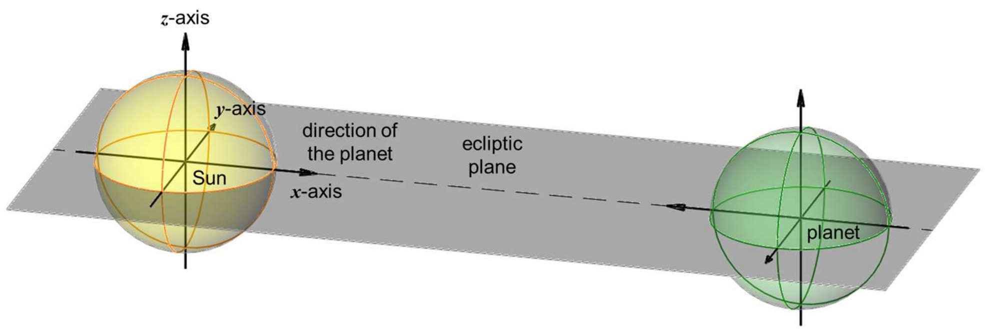
Element	Value
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID-_ORIGIN
Object restrictions	The Sun.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the Sun. 2) The RD Z_AXIS_3D is perpendicular to the ecliptic plane for a specified planet and points in the direction of rotational northwards. 3) The RD X_AXIS_3D is in the ecliptic plane of the specified planet and points towards the planet.
Figures	
References	[HAPG]

Table 7.46 is a directory of heliocentric planet ecliptic ORMs.

Table 7.46 — Heliocentric planet ecliptic ORM directory

ORM label	Published name
HELIO_EARTH_ECLIPTIC	Heliocentric Earth ecliptic

7.5.7 Heliocentric planet equatorial

The *heliocentric planet equatorial OBRS* is specified for a planet in Table 7.47.

Table 7.47 — Heliocentric planet equatorial OBRS

Element	Value
OBRS label	HELIOCENTRIC_PLANET_EQUATORIAL
OBRS code	6
Short name	heliocentric planet equatorial
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	The Sun.

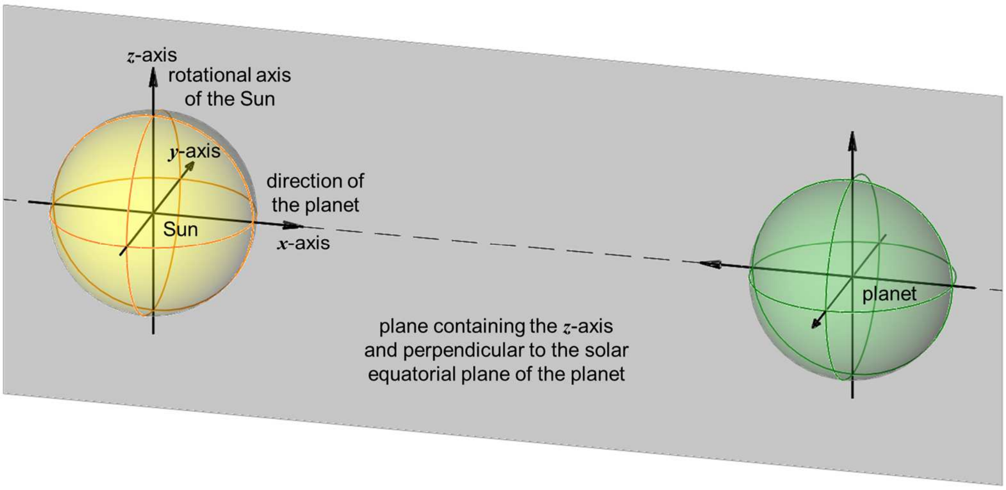
Element	Value
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the Sun. 2) The RD Z_AXIS_3D is perpendicular to the solar equatorial plane of the specified planet and points in the direction of rotational northwards. 3) The RD X_AXIS_3D is perpendicular to the RD Z_AXIS_3D in the plane determined by the RD Z_AXIS_3D and the planet centre and points towards the specified planet.
Figures	
References	[HAPG]

Table 7.48 is a directory of heliocentric planet equatorial ORMs.

Table 7.48 — Heliocentric planet equatorial ORM directory

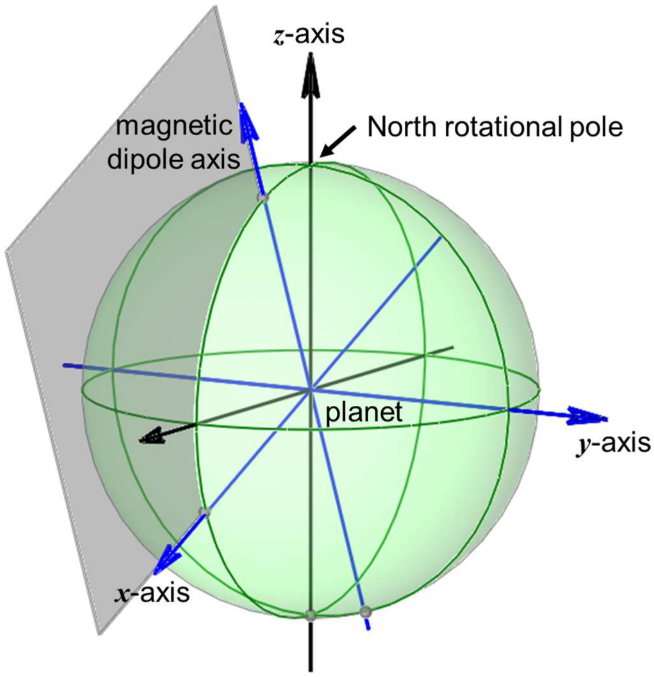
ORM label	Published name
HELIO_EARTH_EQUATORIAL	Heliocentric Earth equatorial

7.5.8 Celestiomagnetic

The *celestiomagnetic OBRS* is specified in Table 7.49. See [BHAV, 3.3.1].

Table 7.49 — Celestiomagnetic OBRS

Element	Value
OBRS label	CELESTIOMAGNETIC
OBRS code	7
Short name	celestiomagnetic
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	A planet or rotating satellite in a solar system with a magnetic dipole axis distinct from its rotational axis.

Element	Value
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD Z_AXIS_3D is parallel to the magnetic dipole axis and points towards magnetic north. 3) The RD X_AXIS_3D is contained in the plane through the origin that is parallel to the dipole and rotational axes, perpendicular to the RD Z_AXIS_3D and pointing away from the dipole northward direction.
Figures	 NOTE The rotational south pole is contained in the x-positive xz-half-plane unless the planet has retrograde rotation. This binding is not applicable to Saturn whose magnetic and rotational poles are not distinguished.
References	[CRUS]

In the case of the Earth, this dynamic ERM is approximated as an Earth-fixed ERM for a five-year epoch. The other celestial objects that have observed magnetic dipoles have object-fixed ORM approximations for the corresponding dynamic ORMs.

Table 7.50 is a directory of the Earth and other object-fixed celestiomagnetic ORM approximations.

Table 7.50 — Celestiomagnetic ORM directory

ORM label	Published name
GEOMAGNETIC_1945_IGRF13	IGRF-13 1945
GEOMAGNETIC_1950_IGRF13	IGRF-13 1950
GEOMAGNETIC_1955_IGRF13	IGRF-13 1955
GEOMAGNETIC_1960_IGRF13	IGRF-13 1960

ORM label	Published name
GEOMAGNETIC_1965_IGRF13	IGRF-13 1965
GEOMAGNETIC_1970_IGRF13	IGRF-13 1970
GEOMAGNETIC_1975_IGRF13	IGRF-13 1975
GEOMAGNETIC_1980_IGRF13	IGRF-13 1980
GEOMAGNETIC_1985_IGRF13	IGRF-13 1985
GEOMAGNETIC_1990_IGRF13	IGRF-13 1990
GEOMAGNETIC_1995_IGRF13	IGRF-13 1995
GEOMAGNETIC_2000_IGRF13	IGRF-13 2000
GEOMAGNETIC_2005_IGRF13	IGRF-13 2005
GEOMAGNETIC_2010_IGRF13	IGRF-13 2010
GEOMAGNETIC_2015_IGRF13	IGRF-13 2015
GEOMAGNETIC_2020_IGRF13	IGRF-13 2020
WORLD_MAGNETIC_MODEL_2010	WMM2010
WORLD_MAGNETIC_MODEL_2015	WMM2015
WORLD_MAGNETIC_MODEL_2020	WMM2020
JUPITER_MAGNETIC_1993	Jupiter magnetic
NEPTUNE_MAGNETIC_1993	Neptune magnetic
SATURN_MAGNETIC_1993	Saturn magnetic
URANUS_MAGNETIC_1993	Uranus magnetic

7.5.9 Solar magnetic ecliptic

The *solar magnetic ecliptic OBRS* is specified in Table 7.51. See [BHAV, 3.3.4].

Table 7.51 — Solar magnetic ecliptic OBRS

Element	Value
OBRS label	SOLAR_MAGNETIC_ECLIPTIC
OBRS code	8
Short name	solar magnetic ecliptic
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN
Object restrictions	A planet in the solar system with a magnetic dipole.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D is in the ecliptic plane of the planet pointing in the direction of the Sun. 3) The RD Z_AXIS_3D is perpendicular to the RD X_AXIS_3D and points in the direction of rotational northwards in the plane determined by the x-axis and the planetary magnetic dipole axis.

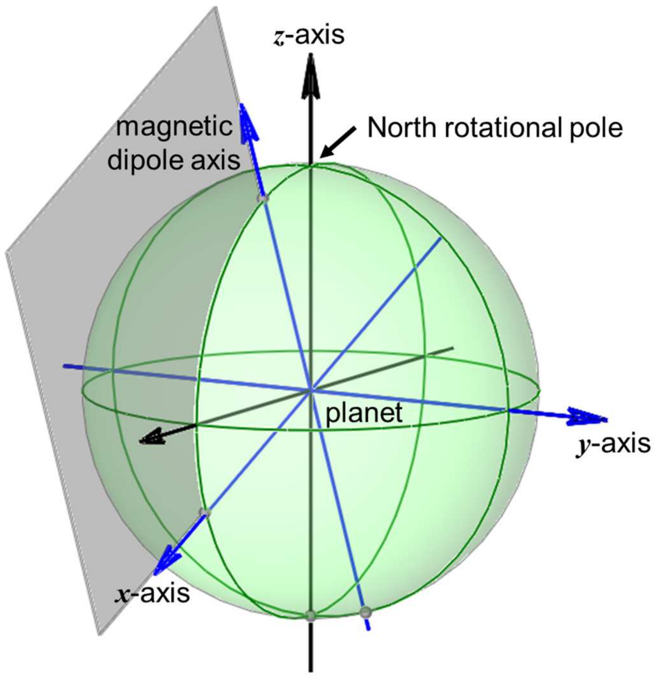
Element	Value
Figures	
References	[CRUS]

Table 7.52 is a directory of the Earth and other planet solar magnetic ecliptic ORMs.

Table 7.52 — Solar magnetic ecliptic ORM directory

ORM label	Published name
EARTH_SOLAR_MAGNETOSPHERIC	Solar magnetospheric
JUPITER_SOLAR_MAG_ECLIPTIC	Jupiter solar magnetic ecliptic

7.5.10 Solar magnetic dipole

The *solar magnetic dipole OBRS* is specified in Table 7.53. See [BHAV, 3.3.5].

Table 7.53 — Solar magnetic dipole OBRS

Element	Value
OBRS label	SOLAR_MAGNETIC_DIPOLE
OBRS code	9
Short name	solar magnetic dipole
Applicable ORMTs	BI_AXIS_ORIGIN_3D SPHERE_ORIGIN OBLATE_ELLIPSOID_ORIGIN

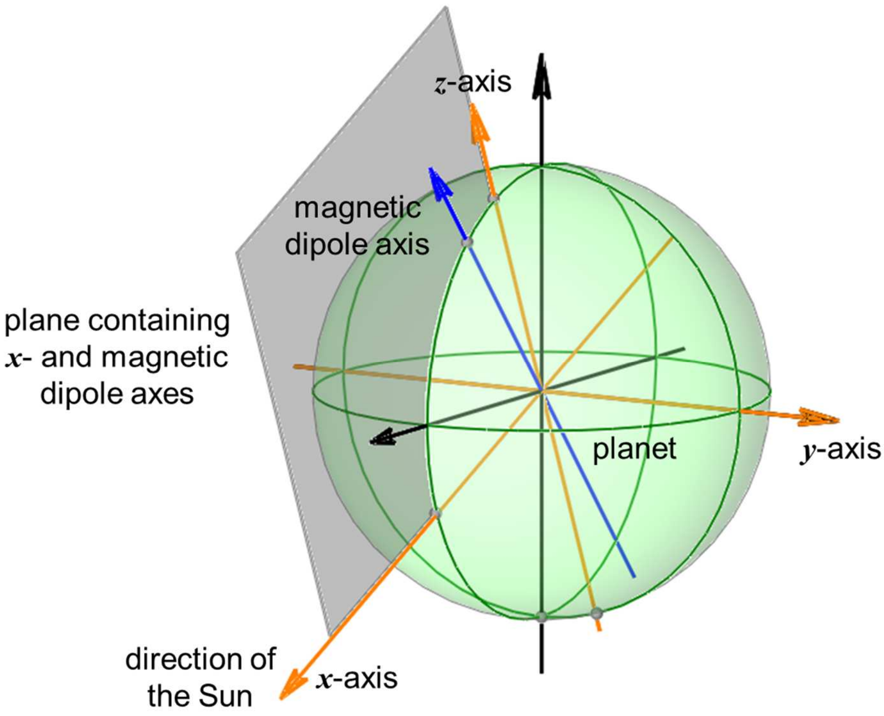
Element	Value
Object restrictions	A planet in the solar system with a magnetic dipole.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD Z_AXIS_3D is parallel to the planetary magnetic dipole axis and points towards magnetic north. 3) The RD X_AXIS_3D is perpendicular to the RD Z_AXIS_3D and pointing towards the Sun in the plane determined by the Sun and the RD Z_AXIS_3D.
Figures	
References	[CRUS], [BHAV]

Table 7.54 is a directory of the Earth and other celestial object solar magnetic dipole ORMs.

Table 7.54 — Solar magnetic dipole ORM directory

ORM label	Published name
EARTH_SOLAR_MAG_DIPOLE	Solar magnetic dipole
JUPITER_SOLAR_MAG_DIPOLE	Jupiter solar magnetic dipole