

**SEDRIS Organization and SRM Editors**  
**Comments on**  
**Spatial Reference Model (SRM)**  
**ISO/IEC 18026 DIS (Revision to Edition 2)**  
**11 October 2012**

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## General Comments

### Throughout

#### **SEDRIS\_G001: Terminology in the Standard**

Review and update all clauses to ensure consistent and proper terminology is being used throughout. Editors have made a number of terminology improvements throughout the standard to improve clarity and consistency. The use of the terms “position”, “position vector”, “location”, “point”, “coordinate”, “distance-preserving”, “length-preserving”, “abstract coordinate system”, “spatial coordinate system”, and “coordinate system” have been reviewed and updated in some cases. Interdependencies throughout the standard are significant and require further careful review to ensure that new, unintended conflicts in the use of terminology have not been introduced. The result of this work should be provided with the next draft version of the standard.

Rationale: Clarity and consistency in use of terms.

### Index

#### **SEDRIS\_G002: Index**

Update the Index for changed page numbers, modified hyperlinks, and changed terminology or new entries resulting from all other changes/comments. Such updates include: changing "object axes" to read "object frame", changing the location (page and hyperlink) for "orthonormal frame" that is now defined in Clause 4 versus Clause 6, and changing "reference axes" to read "reference frames".

Rationale: Consistency in terminology used (and/or updated) throughout the standard.

### Table of contents

#### **SEDRIS\_G003: Table of contents**

Update the Table of contents for changed page numbers, modified hyperlinks, and changed titles or new entries resulting from all other changes/comments. Such updates include: changing the 4.3 title to read "Orthonormal frames, position-space and normal embeddings", changing the Clause 6 title to read "Rotation and orientation", changing the 10.2 title to read "Symbols and terminology", changing the Table 10.1 title to read "Symbols", and changing the Index title to read "Index of defined terms and concepts".

Rationale: Titles changed throughout the standard to better describe specific content.

## Technical Comments

### Clause 4: Concepts

#### SEDRIS\_T001: 4.1, second list item a)

Change to read:

- a) Locations in space are identified by coordinates in a spatial coordinate system. The collection of locations associated with a spatial object of interest, such as the Earth, is termed its object-space (see [4.2](#)). A spatial coordinate system is specified by a spatial reference frame.

Rationale: Clarity and correctness.

#### SEDRIS\_T002: 4.1, second list item f)

Change to read:

- f) A reference datum is a geometric primitive that relates measurements and/or geometric characteristics of object-space to position-space. Object reference models use reference datums to specify a unique normal embedding. An object reference model may also use reference datums to model a geometric aspect of a spatial object. Thus, an oblate ellipsoid reference datum may be used to model the figure of the Earth or other celestial bodies (see [4.4](#) and [Clause 7](#)).

Rationale: Clarity and correctness.

#### SEDRIS\_T003: 4.2

Change the first paragraph to read:

*Object-space* is a universe<sup>1</sup> that is associated with a designated spatial object of interest. Object-space provides the context for spatial concepts including locations, directions, and reference datum binding. A rigid spatial object is assumed to be fixed in its object-space (see [Example 1](#)).

Rationale: Clarity and correctness.

#### SEDRIS\_T004: 4.3

Change the 4.3 title to read "Orthonormal frames, position-space and normal embeddings", and add the following new first paragraph.

"Euclidean space forms a vector space once an origin point has been selected. Each point in Euclidean space may then be associated with the position vector that extends from the origin to the point and has length equal to the Euclidean distance between the origin and the point. Thus, points in space and position vectors with respect to a selected origin may be treated as equivalent concepts. An  $n$ -dimensional vector space with inner product (see [A.2](#)) may also be represented by the vector space of  $n$ -tuples of scalars, provided an orthonormal basis of  $n$  position vectors is selected. The result of selecting an origin and an ordered orthonormal basis is termed an *orthonormal frame*. Every point in  $n$ -dimensional Euclidean space is thus uniquely represented in an orthonormal frame by the linear combination of the basis vectors that is equal to the position vector associated with the point. The  $n$  scalars in the linear combination are represented by a corresponding  $n$ -tuple of scalars in the corresponding basis order."

Rationale: This new paragraph defines *orthonormal frame* as needed for orientation.

**SEDRIS\_T005: 4.3**

Add a new second paragraph:

"A 3D orthonormal frame is termed [right-handed](#) if the vertices of the triangle formed by the basis vector unit points are in clockwise order when viewed from the origin, as defined in [ISO 80000-2](#). In this International Standard, all 3D orthonormal frames shall be right-handed."

Rationale: Right-handed frames are used throughout the standard; Clause 4 is the appropriate place to introduce this definition.

**SEDRIS\_T006: 4.3**

Change the first (now third) paragraph to read:

"*Position-space* is an abstract  $n$ -dimensional orthonormal frame, for  $n = 1, 2,$  or  $3$ . Position-space serves as an abstraction of object-space so that the methods of linear algebra and multivariate calculus may be applied to spatial concepts including the definitions of abstract coordinates systems and the computational aspects of spatial operations."

Rationale: For clarity and correctness after the addition of the new first paragraph that defines *orthonormal frame*. State where and how these mathematical methods are applied.

**SEDRIS\_T007: 4.3**

Change the second (now fourth) paragraph, first sentence, to read:

"A [normal embedding](#) is a distance-preserving function from positions in position-space to points in an object-space of the same dimension."

Rationale: Clarifies the definition by adding the dimensional relationship between position-space and object-space.

**SEDRIS\_T008: 4.3**

Add a new fifth paragraph as follows.

"Given a normal embedding, the origin and basis vectors of the position-space orthonormal frame are mapped to points that form an orthonormal frame in object-space. Conversely, given an orthonormal frame in the object-space, there exists a unique normal embedding that maps the origin and basis vectors of the position-space orthonormal frame to coincide with the origin and basis vectors of the object-space orthonormal frame. An object reference model (see [4.5](#)) implicitly specifies a unique normal embedding in this manner."

Rationale: Clarifies the relationships between normal embeddings, orthonormal frames, and object reference models.

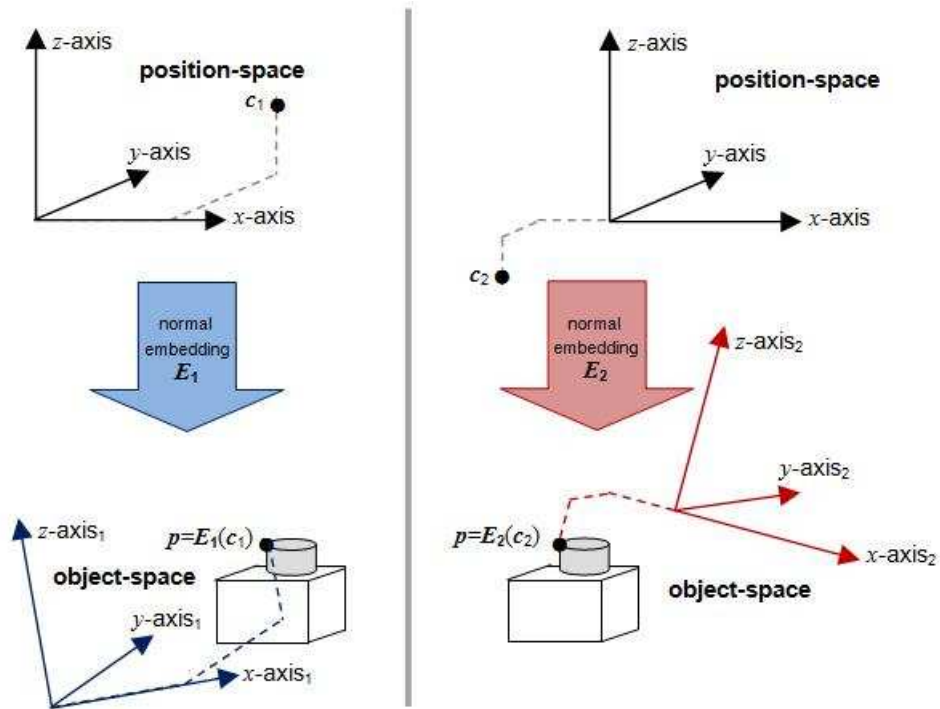
**SEDRIS\_T009: 4.3**

In the fourth (now sixth) paragraph, delete the first sentence (starting: The [origin of a normal embedding](#) is the point ...).

Rationale: The “origin of a normal embedding” is not used again until Clause 7, where it is formally defined, so it is not needed here.

**SEDRIS\_T010: 4.3, Figure 4.2**

Replace the figure with the new improved figure below:



Rationale: More clearly illustrates the relationships among position-space, normal embeddings, and object-spaces.

**SEDRIS\_T011: 4.3**

Change the fifth (now seventh) paragraph to read:

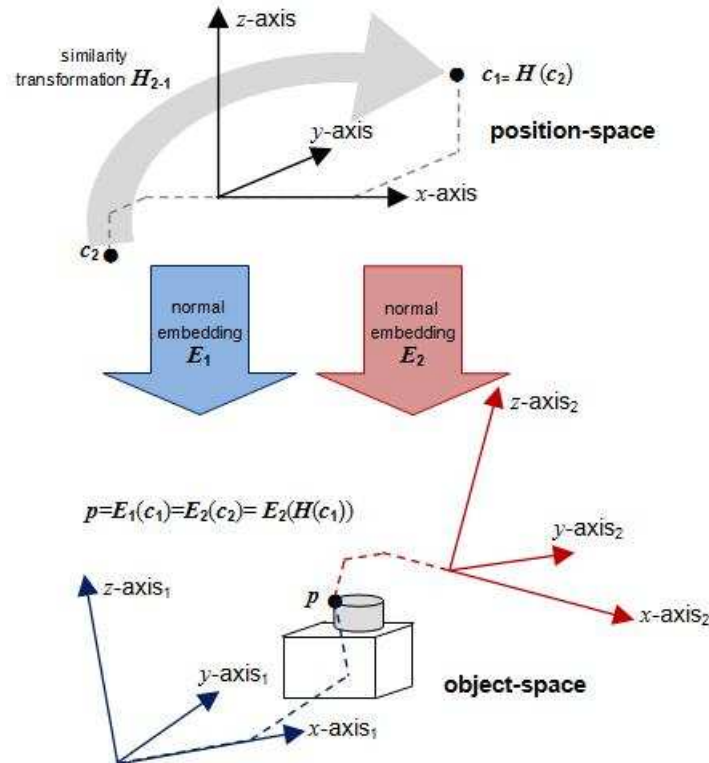
"Any two normal embeddings for the same object-space can be inter-related by a transformation consisting of a translation, a rotation and a scale factor. Such a transformation is termed a [similarity transformation](#). If  $E_1$  and  $E_2$  are two normal embeddings, there is a similarity transformation  $H$  such that  $E_2$  is the composition of  $E_1$  with  $H$ ,  $E_2 = E_1 \circ H$ . This is depicted in [Figure 4.3](#) where  $p = E_1(c_1)$  and  $p = E_2(c_2)$ . The similarity transformation is such that  $c_1 = H(c_2)$  so that  $E_2(c_2) = E_1(H(c_2))$ .

The similarity transformation captures the relationship between two different orthonormal frames in the same object-space, resulting from two different normal embeddings that are based on two different models of the same spatial object (e.g., the Earth). Thus, similarity transformations are used in the transformation of coordinates between two such orthonormal frames."

Rationale: Clarifies the definition of similarity transformation, and its relationship to normal embeddings and orthonormal frames.

**SEDRIS\_T012: 4.3, Figure 4.3**

Insert a new Figure 4.3 (shown below) following the fifth (now seventh) paragraph, and renumber all other figures throughout Clause 4.



Rationale: Illustrates the relationships among position-space, similarity transformations, normal embeddings, and object-spaces.

**SEDRIS\_T013: 4.3, paragraph following the new Figure 4.3**

In the last sentence, change "normal embeddings" to read "orthonormal frames".

Rationale: Uses terminology introduced in the new fifth paragraph of 4.3.

**SEDRIS\_T014: 4.4**

Change the second, third and fourth paragraphs following Figure 4.5 (now Figure 4.6) to read:

"A reference datum binding associates the geometric primitive of a reference datum with a corresponding constructed entity in object-space. A normal embedding may associate the geometric primitive of the same reference datum with its image in object-space. These two methods of associating a reference datum with points in object-space are not necessarily related, nor are the two resulting sets of points necessarily coincident.

A normal embedding of position-space and a reference datum binding are [compatible](#) if the two resulting sets of points are coincident and with the same direction or orientation, as applicable.

In general, a reference datum binding may be compatible with many different normal embeddings. Given a set of two or more bound reference datums, a single normal embedding that is compatible

with each of the bindings may or may not exist. By properly combining multiple reference datum bindings, a unique compatible normal embedding may be specified."

Rationale: Reworded to improve clarity.

**SEDRIS\_T015: 4.5, first paragraph**

Delete the phrase "(right-handed in the 3D case)".

Rationale: Removed as redundant, since all 3D frames in the standard are right-handed.

**SEDRIS\_T016: 4.6.1, first paragraph, second sentence**

Change "The space of these ..." to read "The set of these ...".

Rationale: Corrected terminology.

**SEDRIS\_T017: 4.6.1, Example 4, second sentence**

Change "... the entire space." to read "... the entire Euclidean space."

Rationale: Clarification.

**SEDRIS\_T018: 4.6.2.1, first paragraph**

Change to read:

"There is a requirement to identify time as well as position in environmental representation. Time and position are often used together by an application to describe when a given condition exists or when an object was present at a given location. Furthermore, in dynamic physical systems, the normal embedding that maps position space to an object-space may change over time. As a result, the relationship between coordinates and locations is time-dependent. In such systems, time, and time differences, must be taken into account in order to accurately determine positions and position differences."

Rationale: Improved wording to clarify dynamic relationship between position-space and object-space.

**SEDRIS\_T019: 4.6.3, Example 1**

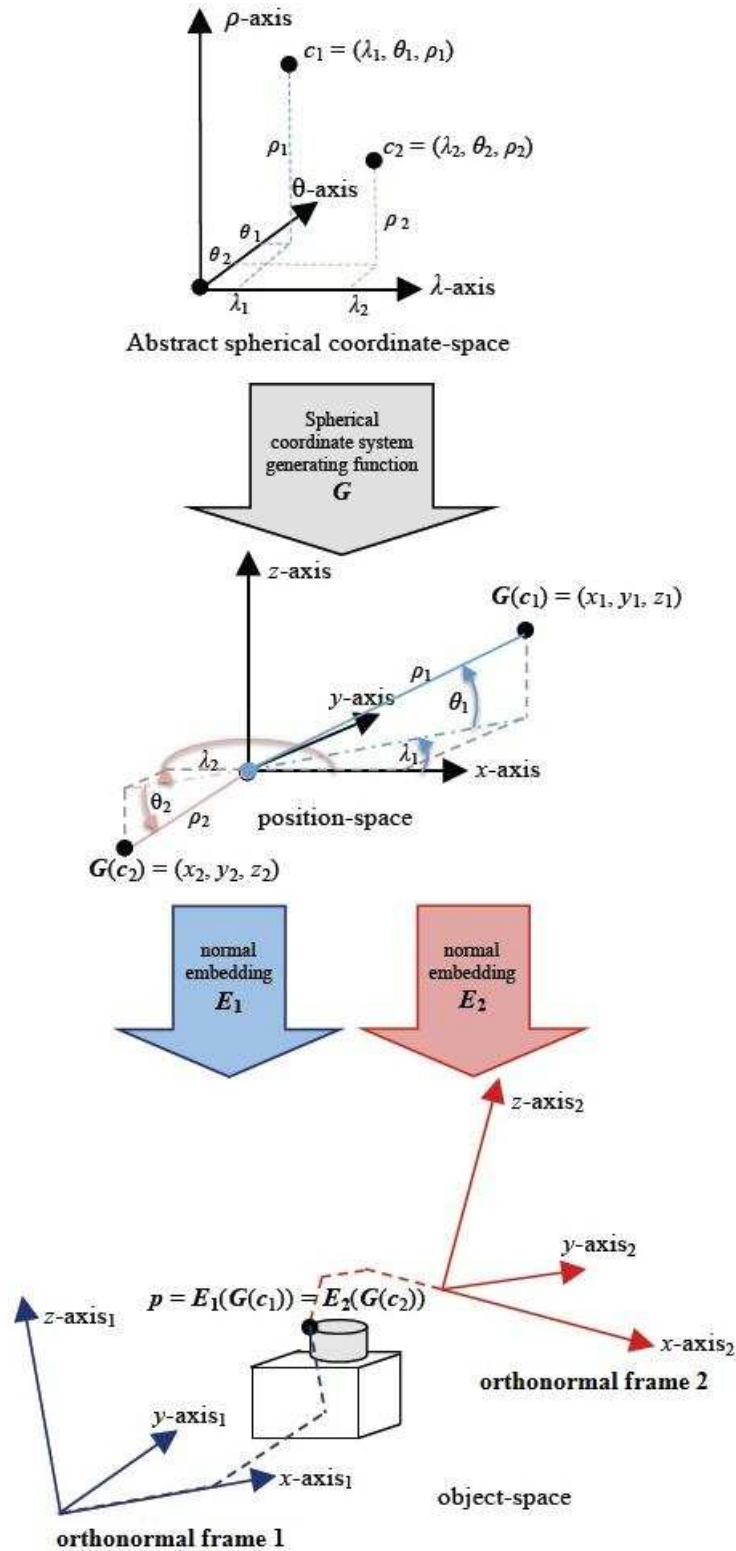
Change to read:

"EXAMPLE 1 Two coordinate tuples in abstract spherical coordinate-space,  $c_1$  and  $c_2$ , are functionally mapped by the spherical coordinate system generating function  $G$  into corresponding position vectors  $G(c_1)$  and  $G(c_2)$  in abstract 3D position-space.  $E_1$  and  $E_2$  are two distinct normal embeddings of 3D position-space into an object-space. A spatial coordinate system is defined by the composition of the generating function  $G$  with the normal embedding  $E_1$ . This composition  $E_1 \circ G$  functionally maps the coordinate  $c_1$  to the point  $p$  in object-space. In this example, the generating function  $G$  is also composed with a second distinct normal embedding  $E_2$  to define a second spatial coordinate system. The composition  $E_2 \circ G$  functionally maps  $c_2$  to the same point  $p$  in the object-space. Thus, the point  $p$  has a spherical coordinate of  $c_1$  in the first spatial coordinate system and a spherical coordinate of  $c_2$  in the second spatial coordinate system. The relationship between the point  $p$  and the coordinates  $c_1$  and  $c_2$  is given by  $p = E_1(G(c_1)) = E_2(G(c_2))$ . See [Figure 4.13](#)."

Rationale: Expanded to address additional relationships shown in new Figure 4.13 (see below).

**SEDRIS\_T020: 4.6.3, Figure 4.12 (now Figure 4.13)**

Replace the current figure with the improved figure below:





Rationale: Improved and expanded figure to better illustrate the relationships between coordinate-space, position-space, and object-space.

### Clause 5: Abstract coordinate systems

#### SEDRIS\_T021: 5.9.12 and 5.9.14 - 5.9.17, Tables 5.18 and 5.20 - 5.23, CS parameters and constraints

Change " $\varepsilon$  : oblate ellipsoid eccentricity ( $\varepsilon \geq 0$ )" to read " $\varepsilon$  : oblate ellipsoid eccentricity ( $0 \leq \varepsilon < 1$ )".

Rationale: Eccentricity bounded above to preserve the integrity of the ellipsoid and avoid computationally degenerate cases.

#### SEDRIS\_T022: 5.9.12 - 5.9.17, Tables 5.18 - 5.23, Domain of the generating function or mapping equations

Change " $-\pi < \lambda \leq \pi$ " to read " $-\pi < \lambda - \lambda_{\text{origin}} \leq \pi$ ".

Rationale: Domain adjusted to match the inverse domain.

#### SEDRIS\_T023: 5.9.13, Table 5.19, Notes

Add the following note: "3) The longitude of origin  $\lambda_{\text{origin}}$  is related to the pole longitude  $\lambda_p$  in [SNYD] by  $\lambda_{\text{origin}} = \lambda_p + \pi/2$ ."

Rationale: New note to make the connection between the SRM formulation and the reference.

#### SEDRIS\_T024: 5.9.13, Table 5.19, References

Change "[SNYD]" to read "[SNYD], [THOM]".

Rationale: Add a reference with the full derivation of the mapping equations.

#### SEDRIS\_T025: 5.9.16, Table 5.22, CS parameters and constraints

Change " $k_0$  : central scale ( $1/2 < k_0 \leq 1$ )" to read " $k_0$  : central scale ( $1/2 \leq k_0 \leq 1$ )".

Rationale: There is no reason to bound  $k_0$  away from  $1/2$ .

#### SEDRIS\_T026: 5.9.16, Table 5.22, Generating function or mapping equations

Change " $E = \frac{a}{b} \left( \frac{1-\varepsilon}{1+\varepsilon} \right)^{\frac{\varepsilon}{2}}$ " to read " $E = \frac{b}{a} \left( \frac{1-\varepsilon}{1+\varepsilon} \right)^{\frac{\varepsilon}{2}}$ ".

Rationale: Parameters  $a$  and  $b$  mistakenly transposed.

## Clause 6: Orientation

### SEDRIS\_T027: 6.8.4, second line following Equation (6.18)

Change " $e_0^2 = \frac{1}{4}(1 + \text{Trace}(\mathbf{R})) = \frac{1}{4}(1 + a_{11} + a_{22} + a_{33})$ " to " $e_0^2 = \frac{1}{4}(1 + \text{Trace}(\mathbf{M})) = \frac{1}{4}(1 + a_{11} + a_{22} + a_{33})$ ".

Rationale: Technical correctness.

### SEDRIS\_T028: Clause 6 - Orientation

Provide a revised Clause 6. Orientation and rotation are highly interrelated topics. The content of Clause 6 must be improved and revised to include a more comprehensive treatment of the subject of rotation. The clause title should also be changed to reflect the treatment of both rotation and orientation topics. Reorganize and rewrite the material in Clause 6, and provide an improved flow of the concepts from the core rotation concepts to the more complex orientation concepts, while maintaining the technical content of any normative concepts that would impact, or that are otherwise used in, other normative clauses of the SRM. Rewrite, reorder, and update the various subclauses to clearly and explicitly address rotation operations, relationships between objects and orthonormal frames, relationships between two or more orthonormal frames, use of consecutive rotation operations, the representation of rotation operations in various forms including matrices, and the use of consistent terms and symbols while including appropriate diagrams to support the topics of rotation and orientation. Make any other changes required for improved consistency and flow of the topics covered in Clause 6.

Rationale: This complex subject has many subtle and interrelated concepts requiring improved exposition, better diagrams, and an improved topic flow that more clearly relates the sub-topics. The Editors have been working on, and continue to work on, a revised draft that is close to achieving these objectives. A preliminary draft is available for any National Body and subject matter experts that wish to review and provide feedback prior to release of the next baseline of the standard.

## Clause 7: Reference datums, embeddings, and object reference models

### SEDRIS\_T029: 7.2

The Editors should investigate using reference datum bindings to construct an orthonormal frame in object-space as a way of simplifying the method of determining a unique normal embedding. Make corresponding changes, if any, in Clause 4 and other clauses.

Rationale: This has the potential to significantly simplify a complex and subtle concept without impacting other concepts in the standard.

### SEDRIS\_T030: 7.3

Once the revision of Clause 6 has been completed, review and update subclause 7.3 as needed to ensure that it remains consistent with the revised Clause 6 with respect to terminology, notation, symbols, and mathematical operations.

Rationale: Correct any inconsistencies introduced by the updating of Clause 6.

## Clause 10: Operations

### SEDRIS\_T031: 10

Once the revision of Clause 6 has been completed, review and update Clause 10 as needed to ensure that it remains consistent with the revised Clause 6 with respect to terminology, notation, symbols, and mathematical operations.

Rationale: Correct any inconsistencies introduced by the updating of Clause 6.

## Clause 11: API

### SEDRIS\_T032: 11.2.9.3.14 and 11.2.9.3.15

Combine the data structures for 2D and 3D ORM transformation parameters (ORM\_Transformation\_2D\_Parameters and ORM\_Transformation\_3D\_Parameters) into a single structure as shown below. Delete 11.2.9.3.15, and replace 11.2.9.3.14 with the following:

#### "11.2.9.3.14 ORM\_Transformation\_Parameters

This variant record data type represents a set of 2D or 3D ORM transformation parameters.

```
ORM_Transformation_Parameters ::= { template_code STT_Code }
{
  [
    IDENTITY:
      identity_parameters      <empty>;
    IDENTITY_2D:
      identity_2d_parameters <empty>;
    TRANSLATE:
      translate_parameters      Translate_3D_Parameters;
    TRANSLATE_2D:
      translate_2d_parameters   Translate_2D_Parameters;
    PV_7_PARAMETER:
      pv_7_parameters          Rotate_Scale_Translate_3D_Parameters;
    CF_7_PARAMETER:
      cf_7_parameters          Rotate_Scale_Translate_3D_Parameters;
    CF_7_PLUS_3_PARAMETER:
      cf_7_plus_3_parameters   Molodensky_Badekas_3D_Parameters;
    ROTATE_SCALE_TRANSLATE:
      rotate_scale_translate_parameters Similarity_3D_Parameters;
    ROTATE_SCALE_TRANSLATE_2D:
      rotate_scale_translate_2d_parameters Similarity_2D_Parameters;
    HOMOGENEOUS_MATRIX_4X4:
      homogeneous_matrix_4x4_parameters Homogeneous_3D_Parameters;
    HOMOGENEOUS_MATRIX_3X3_2D:
      homogeneous_matrix_3x3_2d_parameters Homogeneous_2D_Parameters;
    CF_XYZ_ROTATE_SCALE_TRANSLATE:
      cf_xyz_rotate_scale_translate_parameters
      Rotate_Scale_Translate_3D_Parameters;
    PV_XYZ_ROTATE_TRANSLATE:
      pv_xyz_rotate_translate_parameters Rotate_Translate_3D_Parameters;
```

```

PV_Z_ROTATE_TRANSLATE:
    pv_z_rotate_translate_parameters    Z_Rotate_Translate_3D_Parameters;
CF_Z_ROTATE:
    cf_z_rotate_parameters              Z_Rotate_3D_Parameters;
PV_YZ_ROTATE:
    pv_yz_rotate_parameters             YZ_Rotate_3D_Parameters;
CF_XZ_ROTATE:
    cf_xz_rotate_parameters             XZ_Rotate_3D_Parameters;
]
}"

```

Rationale: Corrects ill-formed variant record structures. `ORM_Transformation_2D_Parameters` and `ORM_Transformation_3D_Parameters` were variant record types using the same selection data type (`STT_Code`), but did not provide a full set of variants for the entirety of the range. Combining `ORM_Transformation_2D_Parameters` and `ORM_Transformation_3D_Parameters` into a single `ORM_Transformation_Parameters` covers each of the variants specified by `STT_Code`.

### **SEDRIS\_T033: 11.3.5.2, Table 11.12**

In the methods `ChangeCoordinate2DSRFObject` and `ChangeCoordinate2DArraySRFObject` of the class `BaseSRF2D`, replace the parameter data type name `ORM_Transformation_2D_Parameters` with [ORM Transformation Parameters](#), and replace the `INVALID_PARAMETERS` error condition with:

```
"INVALID_PARAMETERS if the input h_st parameter values are not valid, or are not of the correct dimension."
```

Update the hyperlinks to reflect the change in the data type name.

Rationale: Update methods parameter types and error conditions to use the revised variant data type `ORM_Transformation_Parameters`.

### **SEDRIS\_T034: 11.3.5.3, Table 11.13**

In the methods `ChangeCoordinate3DSRFObject`, `ChangeCoordinate3DArraySRFObject`, `ChangeDirectionSRFObject`, and `ChangeDirectionArraySRFObject` in the class `BaseSRF3D`, replace the parameter data type name `ORM_Transformation_3D_Parameters` with [ORM Transformation Parameters](#), and replace the `INVALID_PARAMETERS` error condition with:

```
"INVALID_PARAMETERS if the input h_st parameter values are not valid, or are not of the correct dimension."
```

Update the hyperlinks to reflect the change in the data type name.

Rationale: Update method parameter types and error conditions to use the revised variant data type `ORM_Transformation_Parameters`.

### **SEDRIS\_T035: 11**

Once the revision of Clause 6 has been completed, review and update Clause 11 to ensure that it remains consistent with the revised Clauses 6 and 10, or any other clauses, with respect to terminology, notation, symbols, data structures, method descriptions, semantics and signatures.

Rationale: Correct any inconsistencies introduced by the updating of Clause 6.

## **Clause 12: Profiles**

### **SEDRIS\_T036: 12.2, Table 12.1, Error criteria, Accuracy domain template**

Add a second sentence: "Optionally, range constraints on the template and/or ORM parameters".

Rationale: Corresponding to the modified definition of accuracy domain in Clause 14.

### **SEDRIS\_T037: 12.3, Table 12.2, Error criteria, SRFT Label(s) TRANSVERSE\_MERCATOR, MERCATOR, EQUIDISTANT\_CYLINDRICAL, and OBLIQUE\_MERCATOR\_SPHERICAL, Accuracy domain template**

Add "For  $k_0 \geq 0,1$ :" before the existing equations.

Rationale: Parameter constraint added to avoid digital underflows.

## **Clause 14: Conformance**

### **SEDRIS\_T038: 14.2.1, fourth paragraph**

Add a new last sentence: "An accuracy domain template may include constraints on the ranges of the applicable SRFT parameters and/or ORM RD parameters".

Rationale: The current Default Profile includes accuracy domain templates with ORM parameter constraints. This sentence allows for such constraints in an accuracy domain template.

## **Annex A: Mathematical foundations**

### **SEDRIS\_T039: A.11**

Add a new subclause, A.11, as follows:

#### **"A.11 Body-fixed rotations in terms of non-rotating axes**

In the body-fixed convention (defined in [6.3.4](#)), the origin-fixed rotation  $R_n\langle\theta\rangle$  followed by the origin-fixed rotation  $R_m\langle\varphi\rangle$  is the composite operation  $R_{m'}\langle\varphi\rangle \circ R_n\langle\theta\rangle$  where  $m' = R_n\langle\theta\rangle(m)$  is the axis of the second rotation operator after rotation by the first rotation operator.

Define the operator  $\mathbf{R}$  as  $\mathbf{R} = \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ \mathbf{R}_n^{-1}\langle\theta\rangle$ . Then:

$$\begin{aligned}
 \mathbf{R}(m') &= \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ \mathbf{R}_n^{-1}\langle\theta\rangle(m') \\
 &= \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ \mathbf{R}_n^{-1}\langle\theta\rangle(\mathbf{R}_n\langle\theta\rangle(m)) && \text{substitute for } m' \\
 &= \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ (\mathbf{R}_n^{-1}\langle\theta\rangle \circ \mathbf{R}_n\langle\theta\rangle)(m) && \text{replace } \mathbf{R}_n^{-1}\langle\theta\rangle \circ \mathbf{R}_n\langle\theta\rangle = \mathbf{I} \\
 &= \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle(m) && \text{substitute for } \mathbf{R}_m\langle\varphi\rangle(m) \\
 &= \mathbf{R}_n\langle\theta\rangle(m) = m'.
 \end{aligned}$$

Thus  $m'$  is a unit eigenvector for  $\mathbf{R}$ . Euler's rotation theorem (see 6.5) then implies that  $\mathbf{R} = \mathbf{R}_{m'}\langle\pm\varphi\rangle$  with the sign of  $\varphi$  is to be determined. As can be seen from Rodrigues' rotation formula (Equations (6.1) and (6.2)) the limit  $\lim_{\theta \rightarrow 0} \mathbf{R}_n\langle\theta\rangle = \mathbf{I}$  is the identity operator, taking that limit in the expression for  $\mathbf{R}(m')$  yields  $+\varphi$  as the correct sign of the rotation angle. Hence,

$$\mathbf{R}_{m'}\langle\varphi\rangle = \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ \mathbf{R}_n^{-1}\langle\theta\rangle.$$

Substitution of this result in the expression  $\mathbf{R}_{m'}\langle\varphi\rangle \circ \mathbf{R}_n\langle\theta\rangle$  yields:

$$\begin{aligned}
 \mathbf{R}_{m'}\langle\varphi\rangle \circ \mathbf{R}_n\langle\theta\rangle &= (\mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ \mathbf{R}_n^{-1}\langle\theta\rangle) \circ \mathbf{R}_n\langle\theta\rangle && \text{substitute for } \mathbf{R}_{m'}\langle\varphi\rangle \\
 &= \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle \circ (\mathbf{R}_n^{-1}\langle\theta\rangle \circ \mathbf{R}_n\langle\theta\rangle) && \text{replace } \mathbf{R}_n^{-1}\langle\theta\rangle \circ \mathbf{R}_n\langle\theta\rangle = \mathbf{I} \\
 &= \mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle.
 \end{aligned}$$

Thus, the body-fixed composite operator  $\mathbf{R}_{m'}\langle\varphi\rangle \circ \mathbf{R}_n\langle\theta\rangle$  which uses the rotated axis  $m'$  is equal to the composite operator  $\mathbf{R}_n\langle\theta\rangle \circ \mathbf{R}_m\langle\varphi\rangle$  that uses the non-rotated axes  $n$  and  $m$ .

A similar result is true for non-origin-fixed rotations. Let  $\mathbf{R}_{n,t}\langle\theta\rangle$  denote a rotation through angle  $\theta$  about the directed axis  $\{t + \alpha n \mid \alpha \in \mathbf{R}\}$  passing through the position vector  $t$  and parallel to the unit vector  $n$ , and let  $\mathbf{R}_{m,u}\langle\varphi\rangle$  denote a rotation through angle  $\varphi$  about the directed axis  $\{u + \alpha m \mid \alpha \in \mathbf{R}\}$  passing through the position vector  $u$  and parallel to the unit vector  $m$ .

Consider the consecutive rotations  $\mathbf{R}_{n,t}\langle\theta\rangle$  and  $\mathbf{R}_{m,u}\langle\varphi\rangle$  in the body-fixed convention. The first rotation  $\mathbf{R}_{n,t}\langle\theta\rangle$  does not affect the directed axis  $\{t + \alpha n \mid \alpha \in \mathbf{R}\}$ , but does rotate the directed axis  $\{u + \alpha m \mid \alpha \in \mathbf{R}\}$  to a new direction  $m' = \mathbf{R}_{n,t}\langle\theta\rangle(m)$ , and rotates the position vector  $u$  to a new position  $u' = \mathbf{R}_{n,t}\langle\theta\rangle(u)$ . Therefore, the second rotation becomes  $\mathbf{R}_{m',u'}\langle\varphi\rangle$ , yielding the body-fixed composite  $\mathbf{R}_{m',u'}\langle\varphi\rangle \circ \mathbf{R}_{n,t}\langle\theta\rangle$ . The affine operators  $\mathbf{R}_{n,t}\langle\theta\rangle$ ,  $\mathbf{R}_{m,u}\langle\varphi\rangle$  and  $\mathbf{R}_{m',u'}\langle\varphi\rangle$  may be expressed in terms of origin-fixed rotations. Given an arbitrary vector  $r$ :

$$\begin{aligned}
R_{n,t}\langle\theta\rangle(\mathbf{r}) &= R_n\langle\theta\rangle(\mathbf{r}-\mathbf{t})+\mathbf{t} \\
R_{m,u}\langle\varphi\rangle(\mathbf{r}) &= R_m\langle\varphi\rangle(\mathbf{r}-\mathbf{u})+\mathbf{u} \\
R_{m',u'}\langle\varphi\rangle(\mathbf{r}) &= R_{m'}\langle\varphi\rangle(\mathbf{r}-\mathbf{u}')+\mathbf{u}'.
\end{aligned}$$

Substituting, expanding, and simplifying gives:

$$\begin{aligned}
R_{m',u'}\langle\varphi\rangle\circ R_{n,t}\langle\theta\rangle(\mathbf{r}) &= R_{m'}\langle\varphi\rangle\left(R_{n,t}\langle\theta\rangle(\mathbf{r})-\mathbf{u}'\right)+\mathbf{u}' \\
&= R_{m'}\langle\varphi\rangle\left(R_n\langle\theta\rangle(\mathbf{r}-\mathbf{t})+\mathbf{t}-\mathbf{u}'\right)+\mathbf{u}' \\
&= R_{m'}\langle\varphi\rangle\left(R_n\langle\theta\rangle(\mathbf{r}-\mathbf{t})+\chi-\left(R_n\langle\theta\rangle(\mathbf{u}-\mathbf{t})+\chi\right)+R_n\langle\theta\rangle(\mathbf{u}-\mathbf{t})+\mathbf{t}\right) \\
&= R_{m'}\langle\varphi\rangle\left(R_n\langle\theta\rangle(\mathbf{r}-\mathbf{u})\right)+R_n\langle\theta\rangle(\mathbf{u}-\mathbf{t})+\mathbf{t} \\
&= R_{m'}\langle\varphi\rangle\left(R_n\langle\theta\rangle(\mathbf{r}-\mathbf{u})\right)+R_n\langle\theta\rangle(\mathbf{u}-\mathbf{t})+\mathbf{t}
\end{aligned}$$

since  $R_{m'}\langle\varphi\rangle\circ R_n\langle\theta\rangle = R_n\langle\theta\rangle\circ R_{m'}\langle\varphi\rangle$ :

$$\begin{aligned}
&= R_n\langle\theta\rangle\circ R_{m'}\langle\varphi\rangle(\mathbf{r}-\mathbf{u})+R_n\langle\theta\rangle(\mathbf{u}-\mathbf{t})+\mathbf{t} \\
&= R_n\langle\theta\rangle\left(R_{m'}\langle\varphi\rangle(\mathbf{r}-\mathbf{u})+(\mathbf{u}-\mathbf{t})\right)+\mathbf{t} \\
&= R_n\langle\theta\rangle\left(\left(R_{m'}\langle\varphi\rangle(\mathbf{r}-\mathbf{u})+\mathbf{u}\right)-\mathbf{t}\right)+\mathbf{t} \\
&= R_n\langle\theta\rangle\left(R_{m,u}\langle\varphi\rangle(\mathbf{r})-\mathbf{t}\right)+\mathbf{t} \\
&= R_{n,t}\langle\theta\rangle\left(R_{m,u}\langle\varphi\rangle(\mathbf{r})\right) \\
&= R_{n,t}\langle\theta\rangle\circ R_{m,u}\langle\varphi\rangle(\mathbf{r}).
\end{aligned}$$

Hence the general result:

$$R_{m',u'}\langle\varphi\rangle\circ R_{n,t}\langle\theta\rangle = R_{n,t}\langle\theta\rangle\circ R_{m,u}\langle\varphi\rangle.$$

Rationale: Provide results referenced in Clause 6.

## Editorial Comments

### Clause 1: Scope

#### SEDRIS\_E001: 1, first paragraph

Change "position (location)" to read "position".

Rationale: Consistency in terminology.

### Clause 3: Terms, definitions, symbols, and abbreviated terms

#### SEDRIS\_E002: 3.1, NOTE

Delete the NOTE and, instead, include its information in the first paragraph of 3.1 by deleting the first two sentences, and change the title of the Index to "Index of defined terms and concepts".

Rationale: The information serves as an introduction to 3.1, and should not be indicated as a NOTE. Also, the title of the Index should be more expressive of its content.

#### SEDRIS\_E003: 3.2, paragraph preceding Table 3.2

Change "commonly used in" to read "commonly used throughout", and add a second sentence:

"Additional symbols used only in individual clauses are defined within those individual clauses".

Rationale: Make it clear that Clause 3 does not include all symbols used in the document. The use of clause-specific tables is retained to maintain overall usability and readability.

#### SEDRIS\_E004: 3.2, Table 3.2

Change " $R_M$  - radius of curvature in the meridian (see [Table 5.6](#))" to read " $R_M(\varphi)$  - radius of curvature in the meridian at latitude  $\varphi$  (see [Table 5.6](#))", and change " $R_N$  - radius of curvature in the prime vertical (see [Table 5.6](#))" to read " $R_N(\varphi)$  - radius of curvature in the prime vertical at latitude  $\varphi$  (see [Table 5.6](#))".

Rationale: Consistency with the notation used in Table 5.6, where these functions are used.

#### SEDRIS\_E005: Table 3.2 and throughout

Update rotation operator and other notations once the work on Clause 6 and other symbol- and terminology-related changes have been completed. This includes updating  $R_n(\theta)$  and  $\Omega_n(\theta)$  to  $R_n\langle\theta\rangle$  and  $\Omega_n\langle\theta\rangle$ ; ensuring that there are no conflicts such as  $R_N(\varphi)$  - radius of curvature in the prime vertical at latitude  $\varphi$ , and avoid confusing expressions of the form  $R_n(\theta)(p)$ , etc.

Rationale: Improve consistent symbols and terminology in Clause 3 based on other changes throughout the document.



**SEDRIS\_E006: 3.2, Table 3.2**

Delete the "SO(3)" row.

Rationale: This symbol is used only once in the document, and so it need not be included here.

**Clause 4: Concepts**

**SEDRIS\_E007: 4.1, second list item c)**

In the first sentence, delete "its".

Rationale: Grammar.

**SEDRIS\_E008: 4.3**

In the last sentence of the second paragraph, change "In particular, normal embeddings ..." to read "Normal embeddings ...".

Rationale: Readability, too many "in particular"s following each other.

**SEDRIS\_E009: 4.13, third sentence**

Change "... represent and/or process spatial positions ..." to read "... represent and/or process positions ...".

Rationale: Terminology consistency.

**Clause 5: Abstract coordinate systems**

**SEDRIS\_E010: 5.9.4, Table 5.10, Notes**

Change "3) The inverse generation function ..." to read "3) The inverse generating function ...".

Rationale: Terminology clarification.

**SEDRIS\_E011: 5.9.14 and 5.9.16, Tables 5.20 and 5.22, Notes**

Change "3) The CS Generation function ..." to read "3) The CS generating function ...".

Rationale: Terminology clarification.

**SEDRIS\_E012: 5.9.16, Table 5.22, Domain of the generating function or mapping equations**

Change:

$$\begin{array}{ll}
 0 \leq \varphi < \pi/2 \text{ and } (0, \pi/2) & \text{north aspect} \\
 -\pi/2 < \varphi \leq 0 \text{ and } (0, -\pi/2) & \text{south aspect}
 \end{array}$$

to read:

$0 \leq \varphi < \pi/2$  and pole point  $(\lambda, \varphi) = (0, \pi/2)$  north aspect  
 $-\pi/2 < \varphi \leq 0$  and pole point  $(\lambda, \varphi) = (0, -\pi/2)$  south aspect

Rationale: Terminology clarification.

## Clause 7: Reference datums, embeddings, and object reference models

### SEDRIS\_E013: 7.2.5, third paragraph

Change to read:

"If the constructed entity of an RD binding is fixed with respect to object-space, then the RD binding shall be termed an *object-fixed RD binding*. This definition assumes that the constructed entity does not move over time by an amount significant for the accuracy and time scale of an application."

Rationale: Terminology consistency.

### SEDRIS\_E014: 7.3.3, Example, third sentence

Starting in Table 7.24 (through Table 7.28), sequentially renumber the STT code values, beginning with 13 (in Table 7.24), and ending in 17 (in Table 7.28).

Rationale: Two STT tables were previously deleted (with STT codes 13 and 15), however the remaining STT codes were never sequentially renumbered.

### SEDRIS\_E015: 7.3.3, Tables 7.25 and 7.26

Change all " $\omega$ " to read " $\omega_3$ " (three occurrences in each table).

Rationale: Consistency of notation.

## Clause 8: Spatial reference frames

### SEDRIS\_E016: 8.2, Example

Change to read:

"[Figure 8.1](#) illustrates a spatial surface CS bound with a normal embedding of 3D position-space to the 3D object-space. In this illustration, a surface coordinate  $(u, v)$  in coordinate-space is associated to a position-vector  $(x, y, z)$  in position-space. That position then identifies a location in the space of an object via the normal embedding of position-space. In this example, the normal embedding is determined by the selection of an origin and three unit points."

Rationale: Terminology consistency.

## **Clause 10: SRF operations**

### **SEDRIS\_E017: 10.2**

Change title from "Notation and terminology" to read "Symbols and terminology", change the second paragraph to read "The symbols in [Table 10.1](#) are used throughout this clause", and change the title and header row of Table 10.1 to match those of Table 3.2.

Rationale: Clause 10.2 defines "symbols", not "notation", used in Clause 10.

## **Clause 11: API**

### **SEDRIS\_E018: 11.2.7.5**

Change the reference to "Table 7.12" to read "Table 7.30".

Rationale: This reference was not updated after reorganization of Clause 7.

**SEDRIS\_E019: 11.3.5.3, Table 11.13, Abstract method InSRFRegionTestArray, Outputs**  
Change "Status\_array" to read "status\_array".

Rationale: Capitalization error.

**SEDRIS\_E020: 11.3.5.3, Table 11.13, Abstract method InSRFRegionTestArray, Error conditions, Error condition 2)**

Change "INVALID\_COORDINATE if ..." to read "INVALID\_COORDINATE if the [Coordinate3D](#) instance at index in coordinate\_array is (1) not associated with this SRF, or (2) not in the accuracy domain of this SRF".

Rationale: Correct error condition wording.

**SEDRIS\_E021: 11.3.5.3, Table 11.13, Abstract method CreateDirection, Error conditions, Error condition 2)**

Change "INOMPATIBLE\_INPUTS if ..." to read "INCOMPATIBLE\_INPUTS if ...".

Rationale: Spelling error.

**SEDRIS\_E022: 11.3.5.3, Table 11.13, Abstract method ChangeDirectionSRFObject, Semantics, last sentence**

Change "functionally" to read "functionality".

Rationale: Grammatical correctness.

**SEDRIS\_E023: 11.5.9, first sentence**

Change "defined coordinate" to read "defined coordinates".

Rationale: Grammatical correctness.

**SEDRIS\_E024: 11.5.10, first paragraph**

Replace the existing paragraph with:

"This selection data type specifies the RD code associated with a specified RD as defined in [Clause 7](#). [Table 7.3](#) is a directory of RD specifications, each of which includes a code value and a corresponding label. A standardized or registered RD is represented by its RD code."

Rationale: Consistent wording in data type descriptions.

**SEDRIS\_E025: 11.5.11, first paragraph**

Combine the first two sentences to read:

"This selection data type specifies an OBRS code as defined in [Clause 7](#)."

Rationale: Consistent wording in data type descriptions.

**Clause 12: Profiles**

**SEDRIS\_E026: 12.1, second sentence**

Change "... process spatial positions shall be allowed." to read "... process positions in at least one SRF shall be allowed."

Rationale: Terminology consistency.

**SEDRIS\_E027: 12.3, Table 12.2, Error criteria, first two Accuracy domain template rows**

Change "1 000 000 000 m" to read "1 000 000 000m".

Rationale: Consistency of notation.

**SEDRIS\_E028: 12.3, Table 12.2, Error criteria, last two Error bounds rows**

Change " $\varepsilon_D$ " to read " $\varepsilon_D$ ", and change " $\varepsilon_R$ " to read " $\varepsilon_R$ ".

Rationale: Consistency of notation.