5.8.3.3 Scale factors and Map scale.

A thorough discussion of the scale of a map projection must take cognizance of two effects: (1) a typically-small non-linear effect due to the inevitable distortion of representing a curved surface on a flat plane, and (2) a large uniform re-sizing desired to print the map at reasonable size. Both of these are expressed as ratios: $\frac{\text{distance on the projection plane}}{\text{arclength on the ellipsoid}}$. The first ratio, called the *scale factor* varies from point to point. The second ratio, called the *map scale* is a single number for the entire map. The total effect is the multiplication of these two ratios.

EXAMPLE 1. In a 1:50,000 Topographic Line Map, the map scale is $\frac{1}{50{,}000}$. This means that, considering only effect (2), one cm on the map equals $50000\,\text{cm} = 0.5\,\text{km}$ on the ground. At a point some distance from the central meridian of the projection (See Table 5.20), the scale factor is 1.0012. This means that, considering only effect (1), one km on the Earth-sized projection plane equals 1.0012 km on the ground. The combined effect is $1.0012\left(\frac{1}{50000}\right) = \frac{1}{49940}$. In other words, in the vicinity of this particular point, 1 cm on the paper map equals $49940\,\text{cm} = 0.49940\,\text{km}$ on the ground. Rarely does a cartographer use numbers corresponding to the combined effect. It is more meaningful to refer to the separate numbers 1.0012 and $\frac{1}{50000}$.

Two caveats have to be mentioned. In the definition of *scale factor*, it is required that the distance in the projection plane, and the corresponding arclength on the ellipsoid be infinitesimal quantities. This is essential for individual points to own a scale factor. The notion does not attach to curves of finite length.

EXAMPLE 2. In Example 1, in the consideration of effect (1) alone, it was stated that 1 km on the projection plane corresponds to 1.0012 km on the ellipsoid. A better statement is: One meter on the projection plane corresponds to 1.0012 meters on the ellipsoid. An even better statement is: One centimeter on the Earth-sized projection plane corresponds to 1.0012 centimeters on the ellipsoid. Etc.

The second caveat is this. In the definition of scale factor, the direction of the infinitesimal displacement from the given point must be specified. It is enough to specify the scale factor for two independent directions. The scale factor along the meridian is denoted by j. The scale factor along the parallel is denoted by k. If the map projection is conformal, then the scale factor is independent of direction, and $j = k$, and the single scale factor is also called the *point scale*. Scale factors for specific map projections are provided in Clause 10.

EXAMPLE 3 In Example 1, the map projection is conformal. Hence the scale factor 1.0012 is true in every direction from the chosen point.

EXAMPLE 4. A cartographer using an equidistant cylindrical projection (Table 5.33) for a region near the Equator of a spherically modeled Earth, finds that for a point 1 degree North of the equator, $j = 1$ exactly and $k = 1.000152$.

To define what is meant by scale, both concepts were explained. However, the recommendation for most analysts who will define a surface CS based on a map projection, is to have the map scale be 1:1, in other words, no large uniform reduction for printing purposes. The use of the terms "scale factor", "point scale", and "scale factors" may assume that the map scale is 1 : 1. The context will make that clear.